

THREE ESSAYS IN DYNAMIC POLITICAL ECONOMY:  
MIGRATION, WELFARE STATE, AND POVERTY

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All observed government policies must pass through a political process. In many macroeconomic settings, the implemented policies affect the economy not only during the current period, but also the future path of the economy. In this dissertation, I investigate policies pertaining to immigration, redistribution, and poverty reduction.

In the first chapter, I study how politics jointly determine the economy's redistribution and immigration policies. I develop a dynamic political economy model featuring three groups of voters: skilled workers, unskilled workers, and retirees. The model also features both inter- and intra-generational redistribution, resembling a welfare state. To analyze multi-group political economy equilibria, I extend the class of dynamic political games featuring Subgame-perfect Markov as its equilibrium concept. The analysis allows for strategic voting behavior, where voters may vote for a candidate not directly representing their group. Because the policy preference of the unskilled workers is the most intermediate, other groups may choose to side with this policy choice in order to avoid their least preferred candidate. For the unskilled workers, inequality plays a key role in determining the degree of redistribution. Therefore, immigration ultimately affects the generosity of the welfare state by altering the level of inequality in the economy.

The objectives of the second chapter are twofold. First, the chapter tries to understand the relationship between immigration and asset prices. The analysis

reveals that the asset price responds positively to immigration. The immigration's influence goes through four channels: increasing saving, increasing marginal product of capital, decreasing marginal cost of investment, and raising population growth rate. After the preceding analysis, I study how different cohorts will harness these benefits through political interactions. This exercise reveals that the young cohort may have a strategic motive to influence the identity of the decisive voter in the next period to ensure the highest return on their savings in retirement. In addition, the model also predicts that the uncertainty in the population growth rate of the immigrants will lower these immigration quotas.

The last chapter moves away from international policy arena and focuses domestically on escaping a poverty trap. Prior studies conclude that redistribution is a futile policy against this vicious cycle of poverty. I revisit this line of literature and show contrary to this conclusion that redistribution can help the economy escape the poverty trap. I characterize a necessary sequence of lump-sum taxes and transfers and show that this scheme will move the economy out of the poverty trap in finite time regardless of the economy's initial distribution of wealth. Unfortunately, I also show that neither basic democracy nor dictatorship can take the economy there with this policy scheme. The rationale for this is the following. The proposed escape route from poverty requires an economic input from the richer group. However, the shift in the decisive political influence during the path of development, from the hands of the poor to the hands of the rich, will put an end to this pro-poor policy scheme.

## BIOGRAPHICAL SKETCH

Benjarong Suwankiri was born into Suwankiri family in September 1980 as the middle child . He attended Samsaen Kindergarten and Primary School, and enrolled for one year at Rajavinit Mattayom, both located in Bangkok, Thailand. He left Thailand at the age of thirteen, and completed his secondary education at Geelong Grammar School in Victoria, Australia. He obtained his B.A. degree, double majored in Economics and Mathematics with a Minor in General Business, from University of San Francisco in 2002. Prior to his matriculation at Cornell, he was a research intern at the Thailand Development Research Institute. After finishing his first-year course requirements at Cornell, he started pursuing his interest in International Macroeconomics. Then, he changed to International Trade, Economic Theory, Mechanism Design, Public Choice, Political Economics, Social Economics, and Development Economics. After he has tried all these fields, he decided that the field of International Macroeconomics was right for him. While completing his dissertation, he develops additional interests in Labor Economics and Finance. Besides economics, he enjoys reading, cooking ethnic cuisines, and playing variety of sports. He also loves traveling and experiencing cross-cultural activities.

To the giants on whose shoulders we all stand.

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## TABLE OF CONTENTS

Biographical Sketch . . . . .	iii
Dedication . . . . .	iv
Acknowledgements . . . . .	v
Table of Contents . . . . .	viii
List of Figures . . . . .	x
 <b>1 Redistribution Policy and Immigration Policy: A Dynamic Political Economy Analysis</b>	 <b>1</b>
1.1 Introduction . . . . .	1
1.2 The Basic Model . . . . .	5
1.2.1 Demography, Heterogeneity, and Labor . . . . .	7
1.2.2 Fiscal Institution . . . . .	8
1.3 Political Equilibria . . . . .	10
1.3.1 Sincere Voting Outcome . . . . .	11
1.3.2 Strategic Voting Equilibrium . . . . .	16
1.4 Endogenous Wages . . . . .	27
1.4.1 Balanced-Budget Fiscal Institution . . . . .	28
1.4.2 Political Equilibrium . . . . .	29
1.5 Conclusion . . . . .	33
 <b>Bibliography</b>	 <b>35</b>
 <b>2 Immigration and Asset Prices: How Different Cohorts Can Benefit</b>	 <b>38</b>
2.1 Introduction . . . . .	38
2.2 Deterministic Model . . . . .	44
2.2.1 Technologies . . . . .	44
2.2.2 Populations . . . . .	46
2.2.3 Preferences . . . . .	48
2.2.4 Economic Equilibrium . . . . .	49
2.2.5 Political Equilibrium . . . . .	53
2.3 Economy under Uncertainty . . . . .	58
2.3.1 Production . . . . .	59
2.3.2 Populations . . . . .	59
2.3.3 Preference . . . . .	60
2.3.4 Economic Equilibrium . . . . .	61
2.3.5 Political Equilibrium . . . . .	61
2.4 Empirical Epilogue . . . . .	67
2.5 Conclusion . . . . .	69
 <b>Bibliography</b>	 <b>72</b>

<b>3</b>	<b>Are We There Yet? Escaping Poverty Trap with Redistribution</b>	<b>75</b>
3.1	Introduction . . . . .	75
3.2	Galor-Zeira's Model . . . . .	80
3.3	The Redistribution Scheme . . . . .	84
3.4	Political Obstacles to Development . . . . .	94
3.5	Discussion . . . . .	97
3.6	Conclusion . . . . .	100
	<b>Bibliography</b>	<b>103</b>
<b>A</b>	<b>Appendix of Chapter 1</b>	<b>105</b>
A.1	Proofs of the Propositions . . . . .	105
A.2	The Largest Group: Some Accounting . . . . .	113
A.3	Today's Immigration Policies and Tomorrow's Largest Group . . . .	114
<b>B</b>	<b>Appendix to Chapter 2</b>	<b>116</b>
B.1	Proofs . . . . .	116

## LIST OF FIGURES

1.1	Utility in period $t$ of the unskilled worker as a function of the tax rate, $\tau_t$ . . . . .	20
2.1	Fertility Rate (expected number of children per woman). . . . .	39
2.2	Life-expectancy (in years). . . . .	39
2.3	Median Age of Population (in years). . . . .	40
2.4	Old dependency ratio (number of population higher than 65 per 100 of working age group). . . . .	40
2.5	The timing in the deterministic model. . . . .	53
2.6	$\ln(\bar{q})$ , $\ln(q^y)$ , and $\ln(q^o)$ is the natural log level of asset prices in when there is no immigration, the young forms majority, and the old forms majority, respectively. . . . .	58
2.7	The timing of the model with uncertainty. . . . .	62
2.8	Political median (median age of population 18 and over). . . . .	69
3.1	Top diagram shows indirect utilities of skilled ( $s$ ) and unskilled ( $n$ ) with respect to bequest level ( $x$ ). The diagram directly below depicts bequest dynamics. There are 3 steady-states: $\bar{x}_n$ , $g$ , and $\bar{x}_s$ . Their values are given in footnote 6 in the Chapter. . . . .	85

CHAPTER 1

**REDISTRIBUTION POLICY AND IMMIGRATION POLICY: A  
DYNAMIC POLITICAL ECONOMY ANALYSIS**

## **1.1 Introduction**

A welfare state operates both *inter*- and *intra*-generational redistribution. The political system chooses the size of the welfare state to provide benefits to the participants in the economy. In this regard, not only the citizens of the economy contribute to and benefit from the welfare state, immigrants also contribute and benefit as well. The objective of this chapter is to study a joint determination of redistribution policy and immigration policy. In particular, the redistribution policy in mind must have both inter- and intra-generational dimensions to it, resembling a welfare state.

I utilize a two-period-lived, overlapping-generations model. The old cohort retires, while the young cohort works. I divide the young cohort into two skill levels: skilled and unskilled, characterized by the level of wage they earn. The welfare-state system is modeled simply as a proportional tax on income of *all* workers to finance an equal transfer (a demogrant) to *all* agents in the economy in a balanced budget manner. Therefore, everyone benefits from the transfer, but some will bear more costs than others. In addition to the tax-transfer policy, the economy also has immigration. Immigration policies are composed of two variables: one reflects the skill composition of the immigrants and the other captures the immigration volume (per native young population). The skill composition is defined as the proportion of the skilled among the entering immigrants. I characterize subgame-perfect Markov (also known commonly as Markov-perfect) political eco-

economic equilibria in the economy consisting of three groups of voters voting on three policy variables: the tax level, the skill composition of immigrants, and the immigration volume. As a benchmark, I start with sincere voting where the largest group always decides the fate of the policies. However, when having more than two groups participating in the political decision, sincere voting behavior is clearly inadequate. To remedy this shortfall, I then allow for strategic voting among the voters, which opens up the possibility for strategic political coalition.

There have been previous works on the political economy of immigration policies, see for examples Benhabib [1996] and Ortega [2005]. For works on exogenous immigration policy and the political economy of redistribution policy, see Razin, Sadka, and Swagel [2002], and Casarico and Devillanova [2003]. Works that address the joint political economic determination of both types of policy are scant, however. Sand and Razin [2007] pioneers as the first political economy model jointly determining the inter-generational redistribution and immigration. Dolmas and Huffman [2004] and Ortega [2004] analyze similarly the joint determination of intra-generational redistribution and immigration policy in a dynamic political economy model. My analysis amalgamates these two lines of research, allowing for a redistribution across both inter- and intra-generations.

The potential impacts of immigrants on the social security and the welfare state have been studied quite extensively in the literature. See, for example, Smith and Edmonston [1997], which studies different effects of immigrants on the U.S. economy. Most closely related to my dynamic setting are the works by Auerbach and Oreopoulos [1999], Bonin, Raffelhuschen, and Walliser [2000], utilizing a partial equilibrium generational accounting exercise, and Storesletten [2000], employing a dynamic general equilibrium approach. These authors calculate numerically the

costs and benefits of different types of immigrants overtime. In conclusion, they all agree that skilled immigrants have the most potential when it comes to rescuing the frail fiscal system. High-skill immigrants are much more beneficial than low-skill immigrants for the economy, in both short and long term. I incorporate this insight into my model.

On the literature focusing on the sustainability of the social security, for example Cooley and Soares [1999], and Boldrin and Rustichini [2000], and the welfare state in Hassler, Rodriguez-Mora, Storesletten, and Zilibotti [2003], I offer additional insights. Intuitively, the skilled workers will be against redistribution as they bear all the fiscal burden. Therefore, letting in too many skilled immigrants, whose children are also skilled<sup>1</sup>, will lead to an abolishment of the welfare state in the future. Voters, who will be the future beneficiaries of the system, would have an incentive to restrict the amount of skilled immigrants entering the country. This similar channel is fleshed out in Sand and Razin [2007] for the case of young-old conflict, albeit requiring of a negative population growth rate of the natives, and in Dolmas and Huffman [2004], and subsequently in Ortega [2004], for the case of skilled-unskilled conflict. It bears a common feature with models using subgame-perfect Markov equilibrium concept where voters in this period exert influence on the distribution of voters in the next period.

As a by-product from my analysis, I show how the welfare state inline with the preference the unskilled workers would often emerge out of the political process. This adds an interesting angle to the conclusion from Hassler et. al. [20]. When I allow different groups in the economy to vote strategically, both the skilled and

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<sup>1</sup>Black, Devereux, and Salvanes [2005] argues that persistence in education inequality across generations come more primarily from innate ability, rather than education itself. On the contrary, Sacerdote [2002] provides evidence that adoptees going into a high socioeconomic status are likely to achieve higher educational level.

the old voters would vote for the candidate representing the unskilled workers in order to avoid the least-preferred candidate from winning. Hence, even without the unskilled young forming the largest group in the economy, their preferred policies become the most commonly observed in equilibrium. Most notable among these policies is the tax rate, which indirectly determines the size of the welfare state. The unskilled workers will demand more redistribution when the inequality in the economy increases. In addition, the existence of fiscal leakages both to the native beneficiaries (old or unskilled) and the immigrants may lead to a smaller welfare state. These findings confirm the channels previously studied in Razin, Sadka, and Swagel [2002a, 2002b].

Lastly, I also contribute to a rapidly growing field of dynamic political economy, in particular to those employing Markov-perfect as the equilibrium concept. This class of models typically adopts majority voting to resolve conflicts between two groups, hence the group with a dominant size always decides the policy outcome. In this chapter, I allow for political interactions between more than two groups. I build on the work by Besley and Coate [1998] who study a representative democracy in a two-period dynamic environment. In their analysis, the voters take the winning probability of a candidate into their consideration when choosing who to vote for. Although extremely insightful, their approach becomes increasingly entangled with complications arising from massive voting indeterminacy and endogenous candidate selections. These details make their model virtually intractable when extending beyond two periods. To deal with these problems, I take away the candidate endogeneity and force the subgame-perfect Markov property on the voting equilibrium. These produce a much more manageable equilibrium with intuitive explanations.



The chapter proceeds as follow. Section 2 lays out the setup of the benchmark model with fixed wages. In Section 3, I start analyzing the political equilibrium using the model. First, I provide a benchmark by characterizing the political outcome under the assumption of sincere voting. Then I allow strategic-voting behaviors in the latter part of the section. Section 4 endogenizes the wages as determined by the labor markets. I summarize my findings in Section 5 as the conclusion.

## 1.2 The Basic Model

Consider an economy consisting of overlapping generations. Each individual lives for two periods, working in the first period of their lives and retiring when old. The population is divided into two groups according to their exogenously given skills: skilled ( $s$ ) and unskilled ( $u$ ). The old generation does not work. The preference of each generation, both young and old, is given respectively by

$$U^y(c_t^y, l_t^h, c_{t+1}^o) = c_t^y - \frac{\varepsilon(l_t^h)^{1+\frac{1}{\varepsilon}}}{1+\varepsilon} + \beta c_{t+1}^o \quad (1.1)$$

$$U^o(c_t^o) = c_t^o. \quad (1.2)$$

where  $h \in \{s, u\}$ , and  $s$  and  $u$  denote skilled and unskilled labor, respectively. Furthermore,  $y$  and  $o$  correspond to young's and old's utility and consumption,  $\varepsilon$  denotes the elasticity of labor supply, and  $\beta \in (0, 1)$  is the discount factor. Agents in the economy maximize the above utility functions subject to individual's budget constraint. With this preference, equilibrium interest rate equals  $r = \frac{1}{\beta} - 1$  and individuals have no incentive to save, so I take saving to be zero for simplicity.<sup>2</sup>

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<sup>2</sup>Assuming no saving is for pure convenience. With saving, because old individuals do not work the last period of their life, they will consume savings plus any transfer. Through both these channels, the old individuals benefit from immigration. To keep the analysis to a minimal,

This reduces the two groups of old retirees (skilled and unskilled) to just one because they have identical preference irrespective of their skill level.

The consumption good is produced by using the two types of labor as perfect substitute with constant marginal products.<sup>3</sup> The production function is given by

$$Y_t = A_t (w^s L_t^s + w^u L_t^u)$$

where  $u$  and  $s$  denote unskilled and skilled labor, and  $A_t$  denotes Hicks-neutral productivity factor, which is treated as deterministic. Labor markets are competitive, ensuring the wages going to the skilled and unskilled workers are  $\omega_t^s = A_t w^s$  and  $\omega_t^u = A_t w^u$ , respectively. I assume  $w^s > w^u$ .

There is a transfer to everyone in the economy at time  $t$ ,  $T_t$ , financed by a uniform tax across all working individuals,  $\tau_t$ . Following Razin et. al. [2002a], a combination of linear tax and a lump-sum cash grant represents a good approximation of the best egalitarian income tax. In addition to a social security consideration, one can think of the demogrant as the usage of public services by each citizens. I will postpone a detailed description of the the fiscal institution to below. The agents in the economy make economic decisions taking these policy variables as given. Since the old generation has no income, its only source of consumption comes from the transfer. With linear production technology above pinning down the wages for each type of workers, individual's labor supply is given by

$$l_t^h = (A_t w^h (1 - \tau))^\varepsilon, \quad (1.3)$$

---

I will just focus on the costs and benefits in terms of the welfare state. This practice is inline with some recent dynamic models of political economy, for example Hassler et. al. [2003], and Doepke and Zilibotti [2005]. Interested readers in more extensive models with savings are referred to the literature on political economy of social security, for example Cooley and Soares [1999], Boldrin and Rustichini [2000], and Forni [2004].

<sup>3</sup>This simplification, nonetheless, allows me to focus solely on the linkages between the welfare state and immigration, leaving aside any labor market consideration. I consider a model with flexible wages below.

for  $h \in \{s, u\}$ . Putting everything together yield the following indirect utility functions for the young workers and the old retirees, respectively:

$$V^{y,h} = \frac{(A_t w^h (1 - \tau_t))^{1+\varepsilon}}{1 + \varepsilon} + T_t + \beta T_{t+1}$$

$$V^o = T_t,$$

for  $h \in \{s, u\}$ .

### 1.2.1 Demography, Heterogeneity, and Labor

Apart from the tax-transfer policy, the political process also selects immigration policy. This policy consists of two parts: one selecting the volume of immigration, and the other selecting the skill composition. I denote with  $\mu_t$  as the ratio of immigrants to the native-born young population and denote with  $\sigma_t$  the fraction of skilled immigrants entering the country in period  $t$ . Both of these policy variables are restricted to be in a unit interval.

Immigrants have identical preference to the natives. All immigrants come young, so there will never be an entering retired immigrant. I assume all immigrants are naturalized in one period after their entrance.<sup>4</sup> Hence they gain voting power one period after their admittance as old retirees.

I let  $s_t$  denote the fraction of skilled workers in the labor force in period  $t$  (where  $s_0 > 0$ ). The aggregate labor supply of each type of labor is

$$L_t^s = (s_t + \sigma_t \mu_t) N_t l_t^s$$

$$L_t^u = (1 - s_t + (1 - \sigma_t) \mu_t) N_t l_t^u,$$

---

<sup>4</sup>For this model, it is equivalent to roughly 30 years.

where  $N_t$  is the number of native-born young individuals in period  $t$ .

The dynamics of the economy are given by the two population dynamics: one governs the *aggregate* population, while the other governs the *skill* dynamics. Since skills are not endogeneous within the model, I assume for simplicity that the offsprings replicate exactly the skill level of their parents.<sup>5</sup> That is,

$$\begin{aligned} N_{t+1} &= [1 + n + (1 + m)\mu_t] N_t \\ s_{t+1}N_{t+1} &= [(1 + n)s_t + (1 + m)\sigma_t\mu_t] N_t, \end{aligned} \tag{1.4}$$

where  $n$  and  $m$  are growth rate of natives and immigrants, respectively. I restrict  $n, m \in [-1, 1]$  and  $n < m$ . These parameters will be crucial in analyzing any *demographic* benefits of immigration in addition to its *economic* benefits. Combining the two equations together, the skill dynamics can be re-written in a compact form as follows

$$s_{t+1} = \frac{(1 + n)s_t + (1 + m)\sigma_t\mu_t}{1 + n + (1 + m)\mu_t}. \tag{1.5}$$

Equation (1.5) tells me that the next period's fraction of skilled in the labor force will be higher than the present period when the proportion of skilled immigrants in this period is higher than that of the natives,  $\sigma_t > s_t$ . With this setup, immigration is the only way to influence the state variable,  $s_{t+1}$ .

### 1.2.2 Fiscal Institution

I model the fiscal institution as operating with a balanced budget every period. As noted earlier, all workers pay tax at a proportional rate of  $\tau_t$  to their incomes, and

---

<sup>5</sup>Razin, Sadka, and Swagel [2002a] and Casarico and Devillanova [2003] provide a coherent synthesis with endogeneous skill analysis. The first work focuses on the shift in skill distribution of current population, while the latter studies skill-upgrading of future population.

all individuals in the economy benefit equally in the form of per capita transfer,  $T_t$ . There are no other government spending in the economy.

The cohort size of the workers is  $(1 + \mu_t)N_t$  and of retirees is  $(1 + \mu_{t-1})N_{t-1}$ . In period  $t$ , the tax revenue collected from the skilled and unskilled workers is  $\tau_t\{\omega_t^s(s_t + \sigma_t\mu_t)N_t l_t^s + \omega_t^u(1 - s_t + (1 - \sigma_t)\mu_t)N_t l_t^u\}$ . Balanced-budget condition translates algebraically to

$$T_t = \frac{\tau_t((s_t + \sigma_t\mu_t)\omega_t^s l_t^s + (1 - s_t + (1 - \sigma_t)\mu_t)\omega_t^u l_t^u)}{1 + \mu_t + \frac{1 + \mu_{t-1}}{1 + n + \mu_{t-1}(1 + m)}}, \quad (1.6)$$

where the individual's labor supply equations are given above in equation (1.3).  $T_t \left(1 + \mu_t + \frac{1 + \mu_{t-1}}{1 + n + \mu_{t-1}(1 + m)}\right)$  is government spending per *worker*. Note that an increase in immigration or a fall in population growth increases the burden to the working population. I also want to reemphasize here that, due to the nature of this tax and transfer system, the fiscal system provides a channel for redistribution across both *inter*- and *intra*-generation.

I end this subsection with an observation on next period's transfer,  $T_{t+1}$ . It will be increasing in period  $t$ 's skill composition of the immigrants, as long as there are some immigrants. In addition, only when the proportion of the skilled in immigrants is *higher* than in native young will the volume of immigration today helps increase the future transfer. To verify this algebraically, the transfer in period  $t + 1$  is given by,

$$T_{t+1} = \frac{\tau_{t+1}(1 - \tau_{t+1})^\varepsilon A_{t+1}^{1+\varepsilon}}{1 + \mu_{t+1} + \frac{1 + \mu_t}{1 + n + \mu_t(1 + m)}} \left\{ (s_{t+1} + \mu_{t+1}\sigma_{t+1})(w^s)^{1+\varepsilon} + (1 - s_{t+1} + \mu_{t+1}(1 - \sigma_{t+1}))(w^u)^{1+\varepsilon} \right\}.$$

With a little rearrangement, the equation reveals that, as long as  $w^s > w^u$ , next period's transfer increases if the proportion of next period native skilled increases.

Then differentiating the skill dynamic equation (1.5) with respect to  $\sigma_t$  and  $\mu_t$  yields

$$\frac{\partial s_{t+1}}{\partial \sigma_t} = \frac{(1+m)\mu_t}{1+n+\mu_t(1+m)} \text{ and } \frac{\partial s_{t+1}}{\partial \mu_t} = \frac{(1+n)(1+m)(\sigma_t - s_t)}{(1+n+\mu_t(1+m))^2}.$$

The first quantity is always positive as long as there are a positive level of immigration. The second quantity will only be positive when the skilled fraction of immigrants is higher than of natives. Lastly, the quantity in the denominator of  $T_{t+1}$  is always decreasing in  $\mu_t$ . Together, I get the conclusion stated above. Clearly the transfer in period  $t+1$  will not influence the political decision of period  $t$ 's old cohort. However, both types of young individuals will have to take this future benefit of immigrants into their account when vote.

### 1.3 Political Equilibria

In this essay, I focus on subgame-perfect Markov equilibrium. Imagine the economy with three candidates representing each group of voters. With three groups of voters, how I specify the rules of their interaction within the political process is of utmost importance. I first consider the simplest case, where all voters vote *sincerely*. However, the resulting policies will not be an equilibrium under most circumstances, so I refer to them instead as "outcome." In the latter subsection, I relax the restriction on voting behavior to allow for strategic voting and find the political economic equilibrium of the model.

### 1.3.1 Sincere Voting Outcome

In this subsection, I focus on "sincere voting." Sincere voting refers to a voting behavior in which individuals vote according to their *sincere* preference irrespective of the what the final outcome of the political process may be. Consequently, if all voters vote sincerely, plurality rules imply that the largest group in the economy always wins and implement their preferred policies. This is the usual assumption in many political economy models with only two groups of voters. Formally, I define a sincere political outcome as follows.

**Definition 1.** *The policy function  $\Xi_t = \langle \tau_t, \sigma_t, \mu_t \rangle$  constitutes a Subgame-perfect Markov Outcome with Sincere Voting if*

$$\begin{aligned} \Xi_t = \Xi(s_t, \Xi_{t-1}) &= \arg \max_{\tau_t, \sigma_t, \mu_t} V^d(s_t, \Xi_t, \Xi(s_{t+1}, \Xi_t)) \\ \text{s.t. } s_{t+1} &= \frac{(1+n)s_t + (1+m)\sigma_t\mu_t}{1+n+(1+m)\mu_t}, \end{aligned}$$

where  $d \in \{s, u, o\}$  is the identity of the largest group in the economy.

With this restricted voting behavior, the largest group of voters will always have the decisive power, so they will implement their preferred policies in this period. However, the voters realize that policy choice in the next period also matter to them and their choice of policies in this period will affect the choice of the next period's policies through the state variable,  $s_{t+1}$ . Thus the decisive group must choose this period's policies optimally in a forward-looking manner. This consideration proves intractable without any more restriction on the policy functional space. A typical solution is to employ the Markov-perfect restriction that requires the resulting policy function in period  $t+1$  to take the same functional form as the policy function in period  $t$ . The stationarity of the policy function implies that it will be time-independent, except possibly through the state variable.

The following proposition applies this outcome concept and captures what happens if all individuals in this economy vote sincerely.<sup>6</sup>

**Proposition 2** (Sincere-voting Markov Outcome). *The following policy function forms a Subgame-perfect Markov Outcome with Sincere Voting.*

$$\begin{aligned} \tau_t^* &= \begin{cases} 0 & , \text{ if the skilled is the largest} \\ \frac{1 - \frac{1}{J^*}}{1 + \varepsilon - \frac{1}{J^*}} & , \text{ if the unskilled is the largest} \\ \frac{1}{1 + \varepsilon} & , \text{ if the old is the largest} \end{cases} \\ \sigma_t^* &= \begin{cases} 1 & , \text{ if either young is the largest and } s_t \in [0, \frac{1}{1+n}) \\ \hat{\sigma} < \frac{1}{2} & , \text{ if the skilled is the largest and } s_t \geq \frac{1}{1+n} \\ 1 & , \text{ if the old is the largest.} \end{cases} \\ \mu_t^* &= \begin{cases} \frac{1 - (1+n)s_t}{m} & , \text{ if the unskilled is the largest and } \Psi > 0 \text{ or} \\ & \text{if the skilled is the largest and } s_t \in [0, \frac{1}{1+n}) \\ \hat{\mu} < 1 & , \text{ if the skilled is the largest and } s_t \geq \frac{1}{1+n} \\ 1 & , \text{ if the unskilled is the largest and } \Psi \leq 0 \\ & \text{or if the old is the largest.} \end{cases} \end{aligned}$$

where  $J^* = J(\mu_t^*, \sigma_t^*, s_t, \mu_{t-1})$ ,  $\Psi = \Psi(\sigma_t^u = 1, \mu_t^u = \frac{1 - (1+n)s_t}{m})$ ,  $\hat{\sigma}$ , and  $\hat{\mu}$  are given in the appendix.

The proof is in the Appendix. Intuitively, the skilled is the net contributor to the welfare state, while the other two groups are net beneficiaries. If the old cohort is the largest, it wants maximal social security benefits, meaning taxing to the Laffer point ( $\frac{1}{1+\varepsilon}$ ). They also allow the maximal number of skilled immigrants in to the economy because of the tax contribution this generates to the welfare system. When the unskilled group is the largest, it is interesting to note that,

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<sup>6</sup>I provide supplementary analysis on the population accounting of who forms the largest group, and how to influence the next period's identity of largest group in the Appendix A.2 and A.3.



although the unskilled young is a net beneficiary in this welfare state, they are still paying taxes. Hence the preferred tax policy of the unskilled voters is smaller than the Laffer point with a wedge  $\frac{1}{j^*}$ . I will provide further discussions on this deviation factor below. Clearly, the unskilled workers also prefer to let in more skilled immigrants due to their contribution to the welfare state. How many will they let in depends on the function  $\Psi$ , which weighs the future benefits with the cost today. Basically, if the unskilled workers are not forward-looking, it is in their best interest to let in as many skilled immigrants as possible. However, this will lead to no redistribution in the next period because the skilled workers will be the largest. Hence, the function  $\Psi$  is the difference between the benefits they get by being forward-looking and shortsighted.

The skilled natives prefer more skilled immigrants for a different reason from the earlier two groups. They prefer skilled immigrants in this case because this will provide a higher number of skilled native in the *next* period. Thus, if the skilled are forward-looking, they too will prefer more skilled workers in their retirement period. However, they cannot let in too many as this skilled workers in the next period would be the largest group and vote to abolish the welfare state altogether. Note that, given the production function, wages for both workers are constant here. Therefore, the only incentive for more immigrants is to expand the tax base. I will take up the issue relating to labor markets in the next section.

A common feature among political economic models with Markov-perfect concept is the idea that today's voters have the power to influence the identity of future policymakers. Such feature is also prominent in my analysis here. The immigration policy of either young group reflects the fact that they may want to put themselves as the largest old group in the next period. Thus, instead of letting in

too many immigrants, who will give birth to a large new skilled generation, they will want to let in as much as possible before the threshold is crossed. This threshold is  $\frac{1-(1+n)s_t}{m}$ . This strategic motive on immigration policy is previously fleshed out in Sand and Razin [2007]. Letting  $s_t = 1$  gets the result of these authors. There are two differences between my result and the one presented in their paper, however. First, the equilibrium here has a bite even if the population growth rate is *positive*, which is not the case when there are only young and old cohort, as in Sand and Razin [2007], unless the model exhibits a negative population growth rate. Another fundamental difference is that, in order to have some transfer in the economy, the young decisive largest group has a choice of placing the next period's decisive power either in the hand of next period's unskilled or old. So I need to verify an additional condition that it is better for this period's decisive young to choose the old generation next period, which is the case.

When  $s_t \geq \frac{1}{1+n}$ , I have a unique situation (only possible when  $n > 0$ ). In this range of values, the number of skilled is growing too fast to be curbed by reducing immigration volume alone. To ensure that the decisive power lands in the right hand, the skilled voters (who are the largest in this period) must make the unskilled cohort grow to weigh down the growth rate of the skilled workers. This is done by restricting the skill composition as well as the size of total immigration. Empirically, with the population growth rate of the major host countries for immigration like the U.S. and Europe going below 1%, it is unlikely that this case should ever be of much concern.<sup>7</sup>

The tax choice of the unskilled young deserves an independent discussion. In

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<sup>7</sup>Barro and Lee [2000] provides an approximation of the size of the skilled. While Barro and Lee statistics capture those 25 years and above, they also cite OECD statistics which capture age group between 25 and 64. The percentage of this group who received tertiary education or higher in developed countries falls in the range of 15% to 47%.

the work by Razin et. al. [2002a], the authors find that the "fiscal leakage" to the immigrants may result in a lower tax. There are no immigration policy variable in their analysis, and they assume that all immigrants possess lower skill than the natives. Since this increases the burden of the fiscal system, the median voter vote to reduce the size of the welfare state, instead of increasing it. To see such a resemblance of my result with those of these authors, I must first take immigration volume,  $\mu_t$ , and the skill composition,  $\sigma_t$ , as exogeneous. The preferred tax rate of the unskilled native will be

$$\tau_t^u = \frac{1 - \frac{1}{J}}{1 + \varepsilon - \frac{1}{J}},$$

$$\text{where } J = J(s_t, \mu_{t-1}, \mu_t, \sigma_t) = \frac{(s_t + \sigma_t \mu_t) \left( \frac{w_t^s}{w_t^u} \right)^{1+\varepsilon} + 1 - s_t + (1 - \sigma_t) \mu_t}{1 + \mu_t + \frac{1 + \mu_{t-1}}{1 + n + \mu_{t-1}(1+m)}}. \quad (1.7)$$

One can easily verify from these two expressions that  $\frac{\partial \tau_t^u}{\partial \sigma_t} > 0$ , and there exists  $\bar{\sigma}$  such that, for any  $\sigma_t < \bar{\sigma}$ , I have  $\frac{\partial \tau_t^u}{\partial \mu_t} < 0$ . Conversely, for any  $\sigma_t > \bar{\sigma}$ , I would get an expansion of the welfare state, because  $\frac{\partial \tau_t^u}{\partial \mu_t} > 0$ .<sup>8</sup> The inequalities show that higher number of skilled immigrants will prompt a higher demand for intra-generational redistribution. The fiscal leakage channel captures in essence that unskilled immigration creates more fiscal burden, such that the decisive "unskilled" voters would rather have the welfare state shrinks. In addition, an increase in inequality in the economy, reflected in the skill premium ratio  $\frac{w_t^s}{w_t^u}$ , leads to a larger welfare state demanded by the unskilled.

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<sup>8</sup>My use of "shrink" and "expand" should be properly justified. Recall that the tax rate preferred by the unskilled young worker is less than the level that is preferred by the old retirees. The tax rate preferred by the old retirees,  $\tau_t^o = \frac{1}{1+\varepsilon}$  is the Laffer point that attains the maximum welfare size, given immigration policies. Therefore the size of the welfare state is monotonic in the tax rate when  $\tau \in [0, \frac{1}{1+\varepsilon}]$ .

### 1.3.2 Strategic Voting Equilibrium

When there are multiple electorates voting on policies, the assumption of sincere voting is inadequate. After all, people do not always vote for their most sincerely preferred policy, but instead for the *best* policy that most likely will win the election. Such a behavior is referred to as "strategic voting" in the literature. Voting strategically requires all voters to take into their consideration the likelihood that such a policy will be implemented. To allow for strategic voting, I need more apparatuses. I start by discussing some necessary assumptions to make the model tractable.

**Assumption 1.** Voters with the identical preference vote identically.

This assumption simply says that all skilled vote identically, all unskilled vote identically, and all old vote identically. It enables me to forego considering splitting tickets, and vote division within group. Consequently, I only need to look at its representative in order to understand voting behavior of a population group.

**Assumption 2.** Three candidates, one representing each group (skilled, unskilled, and old), submit their proposal for votes.

Unlike the work by Besley and Coate [1997,1998], I do not endogenize the number candidates here, so there are no direct cost of running for office. However, I would imagine that there is an "elective cost" of putting two identical candidates from the same group on the ballot. Alternatively, I can think of the election as a choice of delegation to an individual in the economy to implement the policies. Since there are only three types of individuals in the economy, the delegation will go to one of the three types in the fashion of a citizen-candidates model.<sup>9</sup>

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<sup>9</sup>For seminal works on citizen-candidates model, see Osborne and Slivinski [1996] for sincere voting, and Besley and Coate [1997] for strategic voting setup.

**Assumption 3.** No commitment mechanism for policy implementation.

Under Assumption 2, the implementation power will be in the hands of one of the three types of individuals. Similar to Besley and Coate [1997], I assume that there are no commitment mechanism in the economy. Usually, reputation and reelection motives provide a punishment channel for failing to deliver the promise. Such mechanisms also allow candidates to change and commit to different policy platform to vie for more votes in the election. I assume these are absent from the model. Therefore, the winning candidate cannot credibly commit to implement a policy other than his own ideal policy, representing the demographic group he belongs to.

**Assumption 4.** No abstention.

Voting abstention is a subject of a large literature. I forbid abstention from the model because, under Assumption 1, this reduces the political game to a simple "restricted" median voter game. In addition, without abstention, I am guaranteed to have a policy outcome without a need for an exogenously-defined default policy.

**Voting Decisions.** With an abuse of notation, let the set of three candidates be  $\{s, u, o\}$ , denoting their identity. Then, the decision to vote of any individual must be optimal under the correctly anticipated probability of winning and policy stance of each candidate. Under Assumption 1, I can focus on the decision of a representative voter from each group. Let  $e_t^i \in \{s, u, o\}$  be the vote of the representative voter from type  $i \in \{s, u, o\}$  casted for a candidate. Voting decisions  $\mathbf{e}_t^* = (e_t^{s*}, e_t^{u*}, e_t^{o*})$  is a *voting equilibrium* at time  $t$  if

$$e_t^{i*} = \arg \max \left\{ \sum_{j \in \{s, u, o\}} \mathcal{P}^j(e_t^i, \mathbf{e}_{-it}^*) V^i(\Xi_t^j, \Xi_{t+1}, \mathbf{e}_{t+1}) \mid e_t^i \in \{s, u, o\} \right\} \quad (1.8)$$

for  $i \in \{s, u, o\}$ , where  $\mathcal{P}^j(e_t^i, \mathbf{e}_{-it}^*)$  denotes the probability that candidate  $j \in \{s, u, o\}$  will win given the voting decisions, and  $\mathbf{e}_{-it}^*$  is the optimal voting decision of other groups that is not  $i$ <sup>10</sup>, and  $\Xi_t^j = \langle \tau_t^j, \sigma_t^j, \mu_t^j \rangle$  is the policy vector if candidate  $j$  wins. For instance, under plurality rule,  $\mathcal{P}^j(\mathbf{e}_t^*) = 1$  if  $j$  is the identity of the group with *absolute* majority in the economy.<sup>11</sup> I require each vote casted by each group be a best-response to the votes casted by the other groups. In addition, because Assumption 1 mandates identical voting from individuals of the same characteristics, the representative voter of each group must take into the account the *pivotal* power of their vote.

The voting decision of the old voters can be simplified, since they have no concern for the future,

$$e_t^{o*} = \arg \max \left\{ \sum_{j \in \{s, u, o\}} \mathcal{P}^j(e_t^o, \mathbf{e}_{-ot}^*) V^i(\tau_t^j, \sigma_t^j, \mu_t^j) \mid e_{ot} \in \{s, u, o\} \right\}.$$

Notice that my voting equilibrium requires voters to be forward-looking, rather than retrospective.

**Assumption 5.** Each voting decision is *not* a weakly dominated voting strategy.<sup>12</sup>

Weakly dominated voting strategy typically invites massive indeterminacy into the model, so I try to avoid such a foreseeable pitfall. As an example of such equilibrium, suppose the three groups have the same weight (think one vote per

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<sup>10</sup>The setup for voting equilibrium borrows heavily from Besley and Coate [1997, 1998].

<sup>11</sup>A group is the absolute majority in the economy if its size is more than 50% of the voting population in the economy.

<sup>12</sup>Following Besley and Coate [1997], a voting decision  $e_t^i$  is weakly dominated for  $i$  if there exists  $\tilde{e}^i \in \{s, u, o\}$  such that

$$\sum_{j \in \{s, u, o\}} \mathcal{P}^j(\tilde{e}_t^i, \mathbf{e}_t^{-i}) V^i(\Xi_t^j, \Xi_{t+1}, \mathbf{e}_{t+1}) \geq \sum_{j \in \{s, u, o\}} \mathcal{P}^j(e_t^i, \mathbf{e}_t^{-i}) V^i(\Xi_t^j, \Xi_{t+1}, \mathbf{e}_{t+1})$$

for all  $\mathbf{e}_t^{-i}$  with strict inequality holding for from  $\mathbf{e}_t^{-i}$ .

group, without loss of generality), then if all three cast votes for the worst possible option, this will still satisfy the definition of the voting equilibrium. No group can unilaterally make itself better off. Ruling out weakly dominated voting strategy rules out this undesirable equilibrium.

**Tallying the Votes.** The votes are tallied by adding up the size of each group that have chosen to vote for the candidate. The candidate with the most votes wins the election and gets to implement his ideal set of policies.

Clearly, each individual prefers the ideal policies of their representative candidate. Strategic voting opens up the possibility of voting for someone else that is not their representative candidate to avoid the least favorable policies. For the skilled young, they prefer the least amount of taxes and some immigration for the future. Thus they will prefer the policy choice of the unskilled over the old. As for the old retirees, the more the transfer benefits, the better. Clearly, the unskilled promises some benefits while the skilled promises none, so they would choose the policies of the unskilled over the skilled.

As for the unskilled workers, both rankings are possible: either they prefer the policy choice of the skilled over the old, or vice versa. The parameters of the model will dictate the direction of their votes. The cut-off tax policy,  $\tilde{\tau}$ , is the break-even point for the unskilled voters between preferring some positive tax but receiving transfer (the policies of the old candidate) or no tax at all (the policies of the

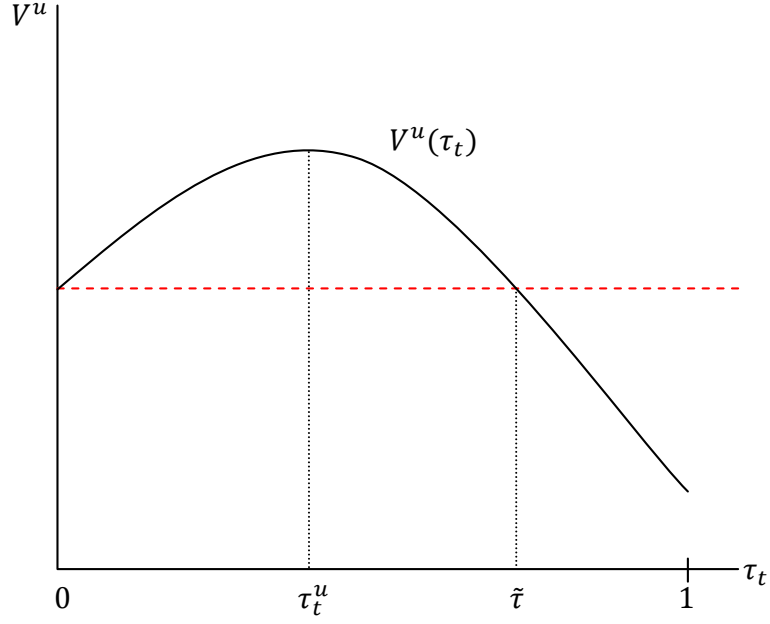


Figure 1.1: Utility in period  $t$  of the unskilled worker as a function of the tax rate,  $\tau_t$ .

skilled candidate). This is depicted in Figure 1.<sup>13</sup> This cut-off tax rate will play an important role for the unskilled young' voting decision.

More generally, the main problem with ranking the utility streams of the voters is due the multiplicity of future equilibria once I extend my model to strategic voting behaviors. This makes it impossible for the (young) voters to get a precise prediction of what will happen as a result of their action today. Even if I could pin down all the relative sizes of all possible payoffs in the next period, multiple

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<sup>13</sup>Formally, this tax level,  $\tilde{\tau}$ , is defined implicitly by the equation

$$\frac{(A_t w^u)^{1+\varepsilon}}{1+\varepsilon} = \frac{(A_t w^u (1-\tilde{\tau}))^{1+\varepsilon}}{1+\varepsilon} + \frac{\tilde{\tau}(1-\tilde{\tau})^\varepsilon A_t^{1+\varepsilon} \left( (s_t + \sigma_t \mu_t) (w^s)^{1+\varepsilon} + (1-s_t + (1-\sigma_t)\mu_t) (w^u)^{1+\varepsilon} \right)}{1 + \mu_t + \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}}.$$

I know that such a tax policy exists, because, take next period's policy as given, the payoff in this period to the unskilled is maximized at its preferred policy and zero at  $\tau = 1$ . Therefore, at some  $\tilde{\tau}$ , the equality will hold.



voting equilibria do not allow a prediction of which equilibrium will be selected in the future. To deal with the problem, I force the voting equilibrium to satisfy a stationary Markov property, similarly to the policy choices in previous subsection. Now, I am ready to define the subgame-perfect Markov political equilibrium under strategic voting.

**Definition 3.** *The policy function  $\Xi_t = \langle \tau_t, \sigma_t, \mu_t \rangle$  with the voting profile  $\mathbf{e}_t^*$  constitutes a Subgame-perfect Markov Equilibrium with Strategic Voting if*

$$\begin{aligned} \Xi_t = \Xi(s_t, \Xi_{t-1}, \mathbf{e}_t^*) &= \arg \max_{\tau_t, \sigma_t, \mu_t} V^d(s_t, \Xi_t, \Xi(s_{t+1}, \Xi_t, \mathbf{e}_t^*)) \\ \text{s.t. } s_{t+1} &= \frac{(1+n)s_t + (1+m)\sigma_t\mu_t}{1+n+(1+m)\mu_t}, \end{aligned}$$

where  $d \in \{s, u, o\}$  is the identity of the the winning candidate, decided by the voting equilibrium  $\mathbf{e}_t^*$  that satisfies equation Assumption 1-5 and the Subgame-perfect Markov property for all  $i \in \{s, u, o\}$ ,

$$e_t^{i*} = \arg \max_{e_t^i \in \{s, u, o\}} \left\{ \sum_{j \in \{s, u, o\}} \mathcal{P}^j(e_t^i, \mathbf{e}_{-it}^*) V^i(\Xi_t^j, \Xi(s_{t+1}, \Xi_t, \mathbf{e}_t^*), \mathbf{e}^*(s_{t+1}, \Xi_t, \mathbf{e}_t^*)) \right\} \quad (1.9)$$

where  $\mathcal{P}^j(e_t^i, \mathbf{e}_{-it}^*)$  denotes the winning probability of the representative candidate  $j \in \{s, u, o\}$  given the voting decisions, and  $\mathbf{e}_{-it}^*$  is the optimal voting decision of other groups that is not  $i$ , and  $\Xi_t^j = \langle \tau_t^j, \sigma_t^j, \mu_t^j \rangle$  is the vector of preferred policy of candidate from group  $j$ .

The stationary Markov-perfect equilibrium defined above introduces the second functional exercise. The first exercise is to find a policy profile that satisfies the usual Markov-perfect definition, as discussed in the Sincere Voting subsection. The second exercise forces the voting decision to be casted on the belief that individuals in the same situation in the next period will vote in exactly the same way. With this property, the voters in this period know exactly how future generations will vote and can evaluate the stream of payoffs accordingly.

Lastly, to keep the analysis to a minimal, I focus on voting equilibria which feature the largest group always voting for its representative candidate. In particular, if a group forms the absolute majority, all votes from this group will go to its representative candidate. This is consistent with the behavior from the equilibrium definition.

For the purpose of my analysis, I assume currently no group holds the absolute majority in the economy. The first equilibrium I look at is dubbed "intermediate policy" because it captures the essence that the preferred policies of the unskilled workers are a compromise from the extremity of the other two groups.

**Proposition 4** (Intermediate Policy Equilibrium). *The following strategy profile forms a Subgame-perfect Markov Equilibrium with Strategic Voting*

$$\begin{aligned}
e_t^{s*} &= \begin{cases} s & , \text{ if } s_t \geq \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \\ u & , \text{ otherwise} \end{cases} \\
e_t^{u*} &= u \\
e_t^{o*} &= \begin{cases} o & , \text{ if } \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \geq \max\{s_t, 1-s_t\} \\ u & , \text{ otherwise} \end{cases}
\end{aligned}$$

and the policies implemented when no group is the absolute majority are

$$\Xi_t = \left\langle \tau_t^* = \frac{1 - \frac{1}{J^*}}{1 + \varepsilon - \frac{1}{J^*}}, \sigma_t^* = 1, \mu_t^* = \frac{2 + n - 2(1+n)s_t}{m} \right\rangle$$

where  $J^* = J(\mu_t^*, \sigma_t^*, s_t, \mu_{t-1})$  is given in the appendix for Proposition 2.<sup>14</sup> The economy can go through different equilibrium paths. 1.) If  $n + m \leq 0$ , the old group is always the absolute majority. Tax rate is at the Laffer point and the economy is fully open to skilled migration. 2.) If  $n + m > 0$ , then the dynamics

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<sup>14</sup>The volume of immigration,  $\mu_t^* = \frac{2+n-2(1+n)s_t}{m}$ , reflects that fact that the threshold value for this variable has been pushed slightly farther. As long as the skilled voters never form the *absolute majority*, the policies in the proposition will always be implemented, even if the skilled voters are the largest group in the economy.

depend on the initial state of the economy,  $s_0$ .  $s_0 \geq \frac{1+\frac{n}{2}}{1+n}$  will have the skilled workers as the absolute majority, and zero tax rate with limited skilled migration.  $\frac{n}{2(1+n)} \geq s_0$  will have the unskilled workers as the absolute majority, with positive tax rate (not Laffer) and some skilled migration.  $n < 0$  will initially place old cohort as the absolute majority with Laffer point tax rate and maximal skilled migration. Otherwise, the policies implemented are given above.

The proof is provided in the appendix. The equilibrium features the unskilled voters always voting for their representative, while the other two groups vote for their respective candidate only if they are the largest group, or for the unskilled candidate otherwise. With these votes, the policies favored by the unskilled young will be implemented almost always, except for when the old or the skilled voters form absolute majority. One notable difference is the policy related to the immigration volume. In period  $t + 1$ , as long as the skilled workers do not form 50% of the voting population, the policies preferred by the unskilled workers will be implemented. To make sure that this is the case, skilled migration is restricted to just the threshold that would have put the skilled voters as the absolute majority in period  $t + 1$ . This level is higher than the restricted volume in Proposition 2.

In the preceding analysis, I let the choice of the skilled workers and the old retirees decide the fate of the the policies. In the following analysis, the unskilled workers consider who they want to vote for. This will depend on how extractive the tax policy preferred by the old is. I call the next equilibrium "Left-wing", because it features a welfare state of the size greater-than-or-equal to that of the intermediate policy equilibrium. This may arise when the tax rate preferred by the old voters is not excessively redistributive.

**Proposition 5** (Left-wing Equilibrium). *When  $\frac{1}{1+\epsilon} \leq \tilde{\tau}$ , the following strategy*

profile forms a Subgame-perfect Markov Equilibrium with Strategic Voting

$$\begin{aligned}
e_t^{s*} &= \begin{cases} s & , \text{ otherwise} \\ u & , \text{ if } \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \geq s_t \geq \frac{1+\frac{n-m}{2}}{1+n} \end{cases} \\
e_t^{u*} &= \begin{cases} u & \begin{cases} , \text{ if } 1-s_t \geq \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \text{ or} \\ \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \geq s_t \geq \frac{1+\frac{n-m}{2}}{1+n} \end{cases} \\ o & , \text{ otherwise} \end{cases} \\
e_t^{o*} &= o
\end{aligned}$$

and the policies implemented when no group is the absolute majority are

$$\Xi_t = \begin{cases} \left\langle \tau_t^* = \frac{1-\frac{1}{J^*}}{1+\varepsilon-\frac{1}{J^*}}, \sigma_t^* = 1, \mu_t^* = \frac{2+n-2(1+n)s_t}{m} \right\rangle & , \text{ if } \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \geq s_t \geq \frac{1+\frac{n-m}{2}}{1+n} \\ \left\langle \tau_t^* = \frac{1}{1+\varepsilon}, \sigma_t^* = 1, \mu_t^* = 1 \right\rangle & , \text{ otherwise} \end{cases}$$

where  $J^* = J(\mu_t^*, \sigma_t^*, s_t, \mu_{t-1})$  comes from Proposition 2 and  $\tilde{\tau}$  is given in the appendix. The economy can go through different equilibrium paths. 1.) If  $n+m \leq 0$ , the old group is always the absolute majority. Tax rate is at the Laffer point and the economy is fully open to skilled migration. 2.) If  $n+m > 0$ , then the dynamics depend on the initial state of the economy,  $s_0$ .  $s_0 \geq \frac{1+\frac{n}{2}}{1+n}$  will have the skilled workers as the absolute majority, and zero tax rate with limited skilled migration.  $\frac{n}{2(1+n)} \geq s_0$  will have the unskilled workers as the absolute majority, with positive tax rate (not Laffer) and some skilled migration.  $n < 0$  will initially place old cohort as the absolute majority with Laffer point tax rate and maximal skilled migration. Otherwise, the policies implemented are given above.

The proof for the proposition is provided in the appendix. When the tax rate preferred by the old voters is not excessively redistributive in the eyes of the unskilled, I could have an equilibrium where the unskilled voters strategically vote for the old candidate to avoid the policies preferred by the skilled voters. This

will be an equilibrium when the size of the skilled is not "too large." Recall that, voting to implement the policies selected by the old candidate leads to opening the economy fully to the skilled immigrants. If the size of the skilled group is currently too large, there is a risk of making the skilled voters the absolute majority in the next period leading to no welfare state in the retirement of this period's workers. The cutoff level before this happens is given by  $\frac{1+\frac{n-m}{2}}{1+n}$ . Therefore, voting for the old will only be compatible with the interest of the unskilled voters when the tax rate is not excessively high and when the size of the skilled is not too large.

To contrast with the above proposition, I turn my attention to the next equilibrium. When the Laffer point is higher than  $\tilde{\tau}$ , the tax rate is read as excessive. In this case, the unskilled voters will instead choose to vote for the skilled over the old candidate. The resulting equilibrium has the size of the welfare state less-than-or-equal to that in the intermediate policy equilibrium, hence I refer to it as "Right-wing."

**Proposition 6** (Right-wing Equilibrium). *When  $\frac{1}{1+\varepsilon} > \tilde{\tau}$ , the following strategy profile forms a Subgame-perfect Markov Equilibrium with Strategic Voting*

$$\begin{aligned} e_t^{s*} &= \begin{cases} s & , \text{ otherwise} \\ u & , \text{ if } 1 - s_t \geq \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \end{cases} \\ e_t^{u*} &= \begin{cases} u & , \text{ otherwise} \\ s & , \text{ if } \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \geq \max\{s_t, 1 - s_t\}. \end{cases} \\ e_t^{o*} &= \begin{cases} o & , \text{ otherwise} \\ u & , \text{ if } s_t \geq \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \end{cases} \end{aligned}$$

and the policies implemented when no group is the absolute majority are

$$\Xi_t = \begin{cases} \left\langle \tau_t^* = 0, \sigma_t^* = 1, \mu_t^* = \frac{2+n-2(1+n)s_t}{m} \right\rangle, & \text{ if } \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \geq \max\{s_t, 1 - s_t\} \\ \left\langle \tau_t^* = \frac{1-\frac{1}{J^*}}{1+\varepsilon-\frac{1}{J^*}}, \sigma_t^* = 1, \mu_t^* = \frac{2+n-2(1+n)s_t}{m} \right\rangle & , \text{ otherwise} \end{cases}$$

where  $J^* = J(\mu_t^*, \sigma_t^*, s_t, \mu_{t-1})$  comes from Proposition 2 and  $\tilde{\tau}$  is given in the appendix. The economy can go through different equilibrium paths. 1.) If  $n + m \leq 0$ , the old group is always the absolute majority. Tax rate is at the Laffer point and the economy is fully open to skilled migration. 2.) If  $n + m > 0$ , then the dynamics depend on the initial state of the economy,  $s_0$ .  $s_0 \geq \frac{1+\frac{n}{2}}{1+n}$  will have the skilled workers as the absolute majority, and zero tax rate with limited skilled migration.  $\frac{n}{2(1+n)} \geq s_0$  will have the unskilled workers as the absolute majority, with positive tax rate (not Laffer) and some skilled migration.  $n < 0$  will initially place old cohort as the absolute majority with Laffer point tax rate and maximal skilled migration. Otherwise, the policies implemented are given above.

I provide the proof in the appendix. When the tax preferred by the old is excessive from the perspective of the unskilled, the political process could implement the policies preferred by the skilled in order to avoid the worst possible outcome. This happens when the old voters constitute the largest group, and the unskilled voters vote strategically for the skilled candidate. In other cases, however, the policies preferred by the unskilled will be implemented, irrespective of the identity of the largest group in the economy.

For my results with multidimensional policies, it is important to note here that the ranking of candidates by individuals allows me to escape the well-known agenda-setting cycle (the "Condorcet paradox"). Such a cycle, which arises when any candidate could be defeated in a pairwise majority voting competition, leads to massive indeterminacy and non-existence of a political equilibrium. The agenda-setting cycle will have a bite if the rankings of the candidates for all groups are unique: no group occupies the same ranked position more than once. However, this does not arise here, because, in all equilibria, some political groups have

a *common* enemy. That is, because they will never vote for the least-preferred candidate (the "common" enemy), the voting cycle breaks down to determinate policies above, albeit their multiplicity. This occurs when voters agree on who is the least-preferred candidate and act together to block her from winning the election. The literature typically avoids the Condorcet paradox by restricting political preferences with some ad hoc assumptions. For my case, the preferences induced from economic assumptions lead to the escape of the Condorcet paradox. For discussions on agenda-setting cycle, see Drazen [2000, pp. 71-72], and Persson and Tabellini [2000, pp. 29-31].

## 1.4 Endogeneous Wages

In this section, I modify the production function to allow for endogeneous wages. I want to allow for interactions between two skill-groups in the production function, but in the most parsimonious way. The production function should capture imperfect substitution between the two groups. Moreover, it should also display any complementarity effects one skill group may have on the other. To stay in line with all these requirements and parsimoniousness, I assume a Cobb-Douglas production function using two skills as inputs to produce a single consumption good. The output is therefore produced by the following production function:

$$Y_t = A_t (L_t^s)^\alpha (L_t^u)^{1-\alpha}$$

where  $A_t$  is the deterministic Hicks-neutral productivity parameter similarly to above, and  $\alpha$  is the share of skilled income. The labor markets are assumed to be competitive, hence the wage paid to the workers equal to marginal product of the

final worker hired, that is

$$w_t^s = \alpha A_t \left( \frac{L_t^u}{L_t^s} \right)^{1-\alpha} \quad \text{and} \quad w_t^u = (1 - \alpha) A_t \left( \frac{L_t^s}{L_t^u} \right)^\alpha.$$

The preference of agents in the economy is the same as above, making the individual labor supply equals to

$$l_t^s = (w_t^s(1 - \tau))^\varepsilon \quad \text{and} \quad l_t^u = (w_t^u(1 - \tau))^\varepsilon,$$

which can be aggregated to give aggregate labor supply in the same manner as above. The two labor markets can be solved simultaneously to find the equilibrium wage for the skilled and unskilled, yielding

$$(w_t^s)^{1+\varepsilon} = \alpha^{1+\varepsilon\alpha} (1 - \alpha)^{\varepsilon(1-\alpha)} A_t^{1+\varepsilon} \left( \frac{1 - s_t + \mu_t(1 - \sigma_t)}{s_t + \mu_t\sigma_t} \right)^{1-\alpha} \quad (1.10)$$

$$(w_t^u)^{1+\varepsilon} = \alpha^{\varepsilon\alpha} (1 - \alpha)^{1+\varepsilon(1-\alpha)} A_t^{1+\varepsilon} \left( \frac{s_t + \mu_t\sigma_t}{1 - s_t + \mu_t(1 - \sigma_t)} \right)^\alpha. \quad (1.11)$$

From the production function, there is really no distinct difference between skilled and unskilled labor force, except from the fact that they are complementary to one another. To resolve this, I assume that  $w_t^s > w_t^u$ , which brings me to the inequality  $\alpha > \frac{s_t + \mu}{1 + \mu}$ . For sufficiency, I can assume  $\alpha > \frac{1 + s_t}{2}$ .<sup>15</sup>

### 1.4.1 Balanced-Budget Fiscal Institution

Similarly to the previous section, the budget must be balanced in all periods. I can make use of equation (1.10) and (1.11) in the labor supply equations to simplify

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<sup>15</sup>Given the nature of exogeneously given skilled distribution and the simple skill dynamics (described in equation (1.5)), this inequality maybe broken in the very long run as  $s_t \nearrow 1$  when  $t \nearrow \infty$ . I could instead assume a Markovian skill dynamics such that there positive probability of being unskilled, depending on family background. For the purpose of my simple positive analysis in period  $t$ , I think that the assumption suffices, as long as I do not attempt to positive analyze the very long run or the steady-state behavior of the political system.



the balanced-budget equation. I have the following

$$\begin{aligned}
T_t & \left( 1 + \mu_t + \frac{1 + \mu_{t-1}}{1 + n + \mu_{t-1}(1 + m)} \right) \\
& = \tau_t ((s_t + \sigma_t \mu_t) w_t^s l_t^s + (1 - s_t + (1 - \sigma_t) \mu_t) w_t^u l_t^u) \\
& = \tau_t (1 - \tau_t)^\varepsilon \alpha^{\varepsilon \alpha} (1 - \alpha)^{\varepsilon(1-\alpha)} A_t^{1+\varepsilon} (s_t + \mu_t \sigma_t)^\alpha (1 - s_t + \mu_t (1 - \sigma_t))^{1-\alpha}.
\end{aligned} \tag{1.12}$$

The last equality follows from substituting for labor supply and wage equations and simplifying some algebra.

The last quantity necessary is the indirect utility of the young (as the indirect utility of the old is simply  $T_t$ ), which is given by

$$V_t^h = \frac{(1 - \tau_t)^{1+\varepsilon}}{1 + \varepsilon} (w^h)^{1+\varepsilon} + T_t + \beta T_{t+1}$$

where  $h \in \{s, u\}$  denotes the skill level of the individual. Wages are given in equation (1.10) and (1.11) for the skilled and unskilled, respectively, and the per-capita transfer is from equation (1.12). Notice right away that, for the young, immigrants affect them through two contemporaneous channels. The first channel is through the labor markets, by either complementarity or substitutability. The second channel comes from financing higher redistribution and transfer in this period. With all these equipped, I can now turn my attention to the political equilibrium.

### 1.4.2 Political Equilibrium

I focus on the sincere-voting political outcome for this section. Recall that for a policy rule to constitute a Subgame-Perfect Markov Outcome with Sincere Voting if the policy rule today takes into the account the policy variables that will be implemented in the future using the same policy rule. In particular, the decisive

voters must consider the effect that today's policy will influence tomorrow's policy through the economy's dynamics.

**Proposition 7** (Sincere-voting Markov Outcome with Endogeneous Wage). *The following strategy profile forms a Subgame-perfect Markov Outcome with Sincere Voting.*

$$\begin{aligned}\tau_t^* &= \begin{cases} 0 & , \text{ if the skilled is the largest} \\ \frac{1 - \frac{1}{K^*}}{1 + \varepsilon - \frac{1}{K^*}} & , \text{ if the unskilled is the largest} \\ \frac{1}{1 + \varepsilon} & , \text{ if the old is the largest} \end{cases} \\ \sigma_t^* &= \begin{cases} \sigma_t^s & , \text{ if the skilled is the largest} \\ 1 & , \text{ if the unskilled is the largest} \\ 1 & , \text{ if the old is the largest.} \end{cases} \\ \mu_t^* &= \begin{cases} \mu_t^s & , \text{ if the skilled is the largest} \\ \frac{1 - (1+n)s_t}{m} & , \text{ if the unskilled is the largest and } \widehat{\Psi} > 0 \\ 1 & , \text{ if the unskilled is the largest and } \widehat{\Psi} \leq 0 \\ & \text{ or if the old is the largest.} \end{cases}\end{aligned}$$

where  $K^* = K(\mu_t^*, \sigma_t^*, s_t, \mu_{t-1})$ ,  $\widehat{\Psi} = \widehat{\Psi}(\sigma_t^u = 1, \mu_t^u = \frac{1 - (1+n)s_t}{m})$ ,  $\sigma^s$ , and  $\mu^s$  are given in the appendix.

I provide the proof in the Appendix. Notice that the proposition looks very similar to the scenario with fixed wages. Nonetheless, there are two marked differences. First, all incentives that drive the preference of agents in the economy come through two channels: wage and transfer. The transfer channel is similar to what I previously discussed under fixed wages. In addition now, through the wage channel, the unskilled workers benefit from complementarity with the skilled workers, lifting up its wage. Hence the unskilled young prefer even more skilled immigration. How many will they let in depends on the function  $\widehat{\Psi}$ , which weighs

future benefits with the cost today. The function  $\widehat{\Psi}$  is the difference between the utility they get by being forward-looking and shortsighted, similarly to the fixed wage setup. The labor market channel is reversed for the skilled workers, who now prefer unskilled immigrants due to their wage complementarity and shun skilled migrants due to their competitive substitution. Nonetheless, the preference of the skilled young is no longer as simple. On the one hand, they prefer unskilled over the skilled immigrant because of the labor market interaction. On the other hand, they want to bring more skilled immigrants whose skilled children would help support the welfare state in the next period. Recall that tax and transfer for this period are zero when the majority is skilled. So any fiscal benefits of immigrants to the skilled young only come next period. Although it seems that this depends crucially on my assumption on the skill dynamics, it is not implausible to imagine a similar consideration arising when children's skill correlates strongly with parent's. Thus even with direct labor market competition, the skilled young may still have incentives to let in skilled immigrants for reasons beyond labor market. As before, the immigration choices reflect the strategy of the younger cohort trying to place its older self as the largest group of voters in the next period.

It is worthwhile to consider the tax preference of the unskilled young in details. I first take immigration volume,  $\mu_t$ , and immigration composition,  $\sigma_t$ , as exogeneous. The preferred tax rate of the unskilled native will be

$$\tau_t^u = \frac{1 - \frac{1}{K}}{1 + \varepsilon - \frac{1}{K}},$$

$$\text{where } K = K(\mu_t, \sigma_t, s_t, \mu_{t-1}) = \frac{1 - s_t + (1 - \sigma_t)\mu_t}{(1 - \alpha) \left( 1 + \mu_t + \frac{1 + \mu_{t-1}}{1 + n + \mu_{t-1}(1 + m)} \right)}.$$

A quick observation reveals that  $\frac{\partial \tau_t^u}{\partial \sigma_t} < 0$  for  $\mu_t > 0$  and  $\frac{\partial \tau_t^u}{\partial s_t} < 0$ . That is, more skilled population in the economy leads to lowering the tax burden. These result has to be contrast with the scenario with fixed wage. Under fixed wage,

more number of skilled means more intra-generational redistribution. With flexible wages, the unskilled majority needs to consider more than just redistribution. Any increase in the skilled composition increases both the unskilled wage and transfer, while lowering the skilled wage. Nonetheless, the benefit to the unskilled cohort through labor market is private, while through welfare-state is shared among all citizens. By lowering tax rate, the unskilled young gets to enjoy the benefit of higher wages from complementarity with skilled, albeit lower transfer. In this more flexible-wage framework, a higher number of skilled workers also *reduces* the tax rate preferred by the unskilled. Hence, under my specification, the larger skill composition in the economy (both of the native-born and the immigrants) *automatically* lowers wage differential in the economy, hence the need for intra-generational transfer.

Inequality still plays an important role in the tax preference of the unskilled, but through a different parameter. When,  $1 - \alpha$ , the share of income going to the unskilled workers falls, tax rate rises to redistribute more heavily. This can be thought along the line of higher inequality leads to more redistribution.<sup>16</sup> The fiscal leakage channel also reflects these automatic correction in wage differential across the two working groups. It can be shown that there exists a cut-off composition,  $\tilde{\sigma}$ , such that, for any  $\sigma_t < \tilde{\sigma}$ , I have an expansion of the welfare state ( $\frac{\partial \tau_t^u}{\partial \mu_t} > 0$ ), and any  $\sigma_t > \tilde{\sigma}$ , I would get the opposite ( $\frac{\partial \tau_t^u}{\partial \mu_t} < 0$ ). In words, more unskilled composition of the immigrants creates more desire for redistribution, enlarging the size of the welfare state.

Let me discuss briefly about the political equilibria with strategic voting behavior under flexible wages. Recall that, under fixed wages, political coalitions are

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<sup>16</sup>In fact, more accurately, the relevant measure of inequality for this expression is share of the income over the share of population,  $\frac{(1-\alpha)}{1-s_t+(1-\sigma_t)\mu_t}$ . As this rises, the demand for redistribution falls.

formed either between the skilled and the unskilled workers or between the old retirees and the unskilled workers. These formations continue to be true under the case of flexible wages. The skilled workers prefer the least number of skilled immigrants and the lowest level of tax. Thus they will prefer the policy choice of the unskilled over the old candidate. On the contrary, the old retirees want the highest degree of skilled immigrants and the maximal size of the welfare state. Hence they will prefer the policy choice of the unskilled over the skilled candidate. Which candidate the unskilled workers decide to vote for depends on the gain from the labor and the degree of redistribution of the Laffer point. All in all, the equilibria will look almost identical to the one described in the preceding section.

## 1.5 Conclusion

To address the linkage between redistribution policy and immigration policy, I build a dynamic political economy model featuring three groups of voters: skilled workers, unskilled workers, and retirees. The model features both *inter*- and *intra*-generational redistribution, resembling a welfare state. The skilled workers are net contributors, while the unskilled workers and old retirees are net beneficiaries. However, skilled immigrants also increase the political threat against the welfare state because their offsprings, who are skilled, will vote against it in the next period. Voters who will be net beneficiaries from the welfare state may have an incentive to restrict the number of skilled immigrants such that the welfare state continues to operate in the future. When the skilled cohort procreates rapidly (both due to high population growth, and large initial size), it may be necessary to bring in unskilled immigrants to counter balance the expanding size of the skilled group.

I extend the class of dynamic political economy models using Subgame-perfect Markov as equilibrium concept to allow for strategic voting behavior. This is necessary when the economy has more than two types of electorates. It allows voters to cast vote for a candidate not directly representing their sincere preference to defeat their least preferred candidate. I find that the policies preference of the unskilled natives are likely to prevail over that of the others. This is true irrespective of the identity of the largest group of voters in the economy. The policy preference of the unskilled is the most intermediate, making it an attractive locus to avoid the worst possible policies. Because unskilled workers both pay taxes to and benefit from the welfare state, inequality plays a key role in determining the size of the tax rate and redistribution. Therefore, immigration ultimately affects the generosity of the welfare state by affecting the level of inequality in the economy.

Lastly, there is a large literature studying how social security would emerge as a political equilibrium when the young generation forms the majority in the economy. Much of the literature conclude that the necessary ingredient for such an emergence is the aging of the economy and/or some other equilibrium mechanisms that imposes punitive cost if the social security system were to stop (see for example, Cooley & Soares [1999], and Boldrin & Rustichini [2000]). My results here suggest an additional perspective. I find that the social security system needs not arise only from a conflict between young and old voters. If the welfare state provides a vehicle for redistribution beyond just one dimension across generations, a positive level of social security could arise as a by-product of a strategic political coalition.

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CHAPTER 2

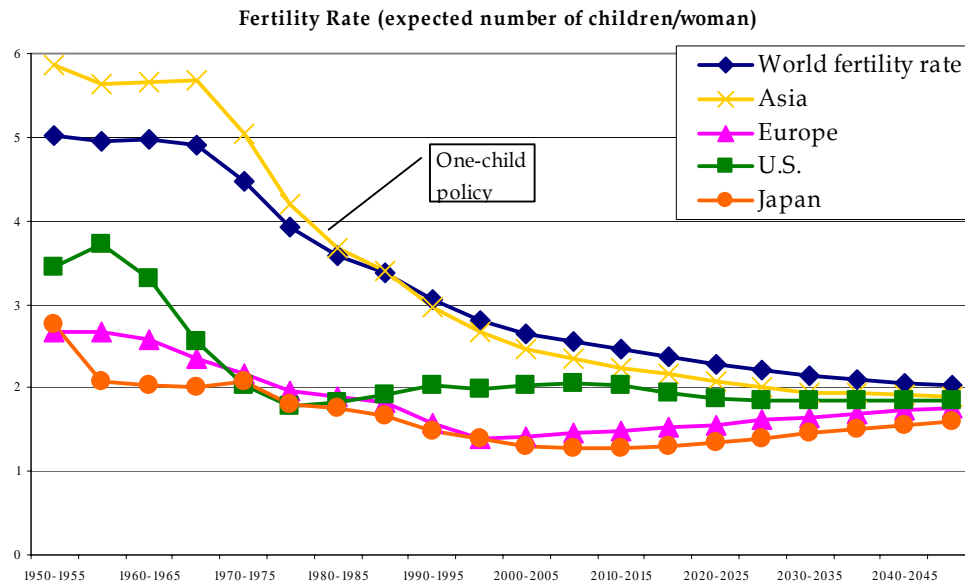
**IMMIGRATION AND ASSET PRICES: HOW DIFFERENT  
COHORTS CAN BENEFIT**

## **2.1 Introduction**

Two recent policy-related debates (which are still on-going), namely, immigration into the U.S. and consequences of aging demography on social security have led me to consider a mutual viewpoint: how would a large aging political cohort think about immigration into the country. The model for answering questions about change in demographic structures have been around for a long time, but only until recently have economists asked questions of causal relationships between demographic structures and economic outcomes. This partially reflects the need for answers to questions raised by the babyboomers about the babyboomers.

High fertility rate during the late 1950's and early 1960's creates the largest demographic cohort that we know as the "babyboomers" (see Figure 2.1). Half a decade later, this same large cohort is on the verge of retirement. With falling fertility rate and rising life-expectancy (Figure 2.2 and 2.3), the whole world seems to age along the side of the babyboomers. All these contributed to an upward trend in the old-age dependency ratio, as presented in Figure 2.4, putting more and more pressure on the working-age group. The upward trend in the dependency ratio is projected to continue well into the middle of the twenty-first century (projections by the United Nations Population Division).

The objectives of this chapter are twofold. First, I try to understand the relationship between immigration and asset prices. The analysis reveals that asset



Source: UN Population Division

Figure 2.1: Fertility Rate (expected number of children per woman).

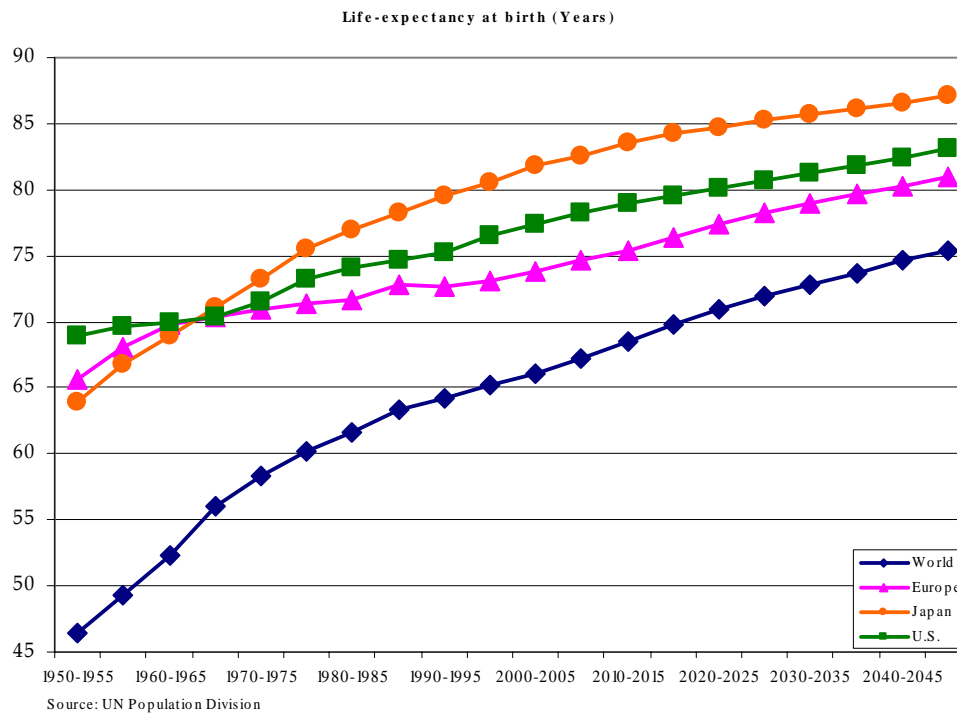


Figure 2.2: Life-expectancy (in years).

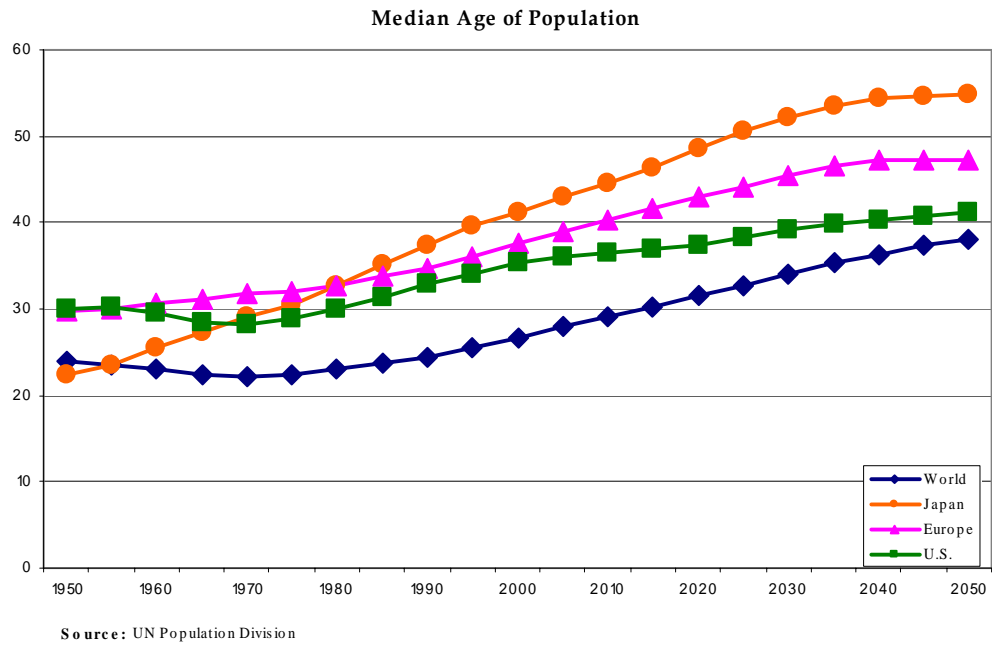


Figure 2.3: Median Age of Population (in years).

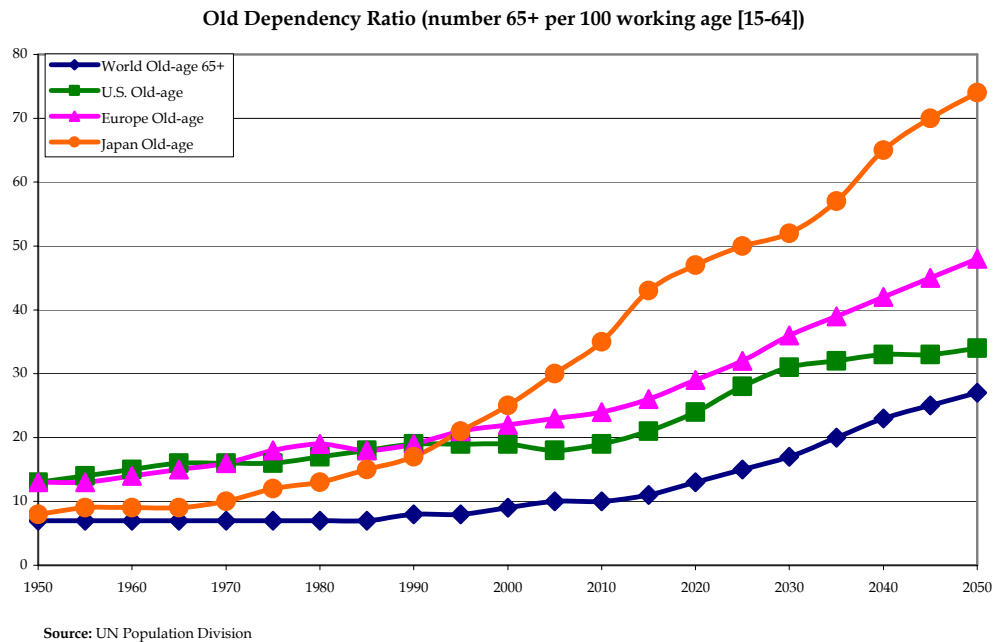


Figure 2.4: Old dependency ratio (number of population higher than 65 per 100 of working age group).

prices respond positively to immigration. It also predicts that immigration in the present period positively affect asset prices in this period and the next. Then, once I have found these effects, I ask how will different cohorts harness these benefits through political interactions. I characterize analytically the political economic equilibrium level of immigration, using the Markov-perfect equilibrium concept pioneered by Krusell and Rios-Rull [1996], which is later solved for closed-form solutions for the first time in Hassler, Rodriguez-Mora, Storesletten, and Zilibotti [2003]. The concept has wide applicability and my application to immigration is not the first. Sand and Razin [2007], in particular, has used the equilibrium concept to shed lights on the question of social security sustainability and the role of immigration. Ortega [2004, 2005] have also employed the concept to deal with the dynamic choice of immigration policy.

The implications from this chapter with respect to aging demography contribute to a couple lines of literatures. The first line focuses on the cost and benefits of immigration. For some comprehensive treatments on economic impact of immigration, see work commissioned by National Research Council [1997] and Simon [1999]. A particular subset of these literatures under my consideration is sustainability of social security system and migration. As the media have popularized, the PAYG social security system is on the brink of insolvency because the world's dependency ratio continues to rise. Many researchers and thinktanks have called for an overhaul of the entire system. While the discussion remains in limelight, another group of academic researchers consider a shorter-term solution to the problem: immigration. Examples of these contributions are Storesletten [2000], Razin and Sadka [2001], and Sand and Razin [2007]. Each of these studies approaches the problem from slightly different angles, but ultimately reach the same conclusion: young immigrants could help lowering the burden of tax to

support the withering social security. Other literatures on political economy of immigration have similarly focused on expanding the tax base using this line of arguments. In this chapter, I offer another viewpoint on the benefits of immigration through their possible influence on asset prices. The pension funds are invested in some form of assets, whether they'll include stocks, bonds, or simply interest-bearing accounts. Hence if we could find a way to sustain rate of returns on these investment, we will have succeeded, at least partially, in releasing the pressures of the current system. Although I do not claim to have a complete model to capture such a complex social security investment process (as done in Abel [2003]), but certainly, I have brought a new light to the issue. The point about increasing capital returns to native citizens from immigration as been brought into to discussion before in Berry and Soligo [1969], however, their discussions and any subsequent works fail to look the "valuation" effects of immigration on asset prices. Other economic literatures on immigration focus entirely on labor market and fiscal burden, but rarely on other issues (recent exceptions are Saiz [2003, 2007], and Lach [2007]).

My analysis also has another implication. There is a big literature focusing on the stock market meltdown and the effect of babyboomers' retirement. The controversy reaches its peak with debates from both sides, one supporting the argument for a major impact on the stock market (See, for example, Abel [2001, 2003], and Geanakoplos Et Al [2004]) while the other believing in less of an effect (see for example Poterba [2001, 2004], and Brooks [2002, 2006]). On the one hand, those strongly believe in the meltdown theory argue that the retirement of this unusually large size babyboom cohort will cause a plunge in the value of the stock markets. Earlier, this cohort has been saving on different assets pushing their prices higher. As this large cohort retires and sells its lifetime assets to the smaller

cohort, the supply exceeds the demand and asset prices fall. Such perspective relies heavily on the life-cycle pattern of consumption. On the other hand, many empirical works fail to find strong empirical linkages between demography and asset returns. As Brooks [2006] points out, even if one finds empirical support for the theory, it appears that many studies suffer seriously from spurious regression: an external factor that affects both asset prices and fertility rates. In addition, some evidence also suggests that households may not run down their accumulated asset in retirements, as claimed by the earlier group whose logic is based on the life-cycle hypothesis. In fact, Poterba [2001] and Brooks [2006] find that the survey evidence suggests the reverse: households save more after retirement. My result steps right inside of the circle. The model builds on the standard overlapping-generations framework, so it is subjected to the scrutiny put forth against such modeling techniques for this literature. However, even with such set up, I show that asset prices may go up in response to this demographic shift. Given that the babyboomers, a large cohort, will likely face unattractive asset markets in the future, this cohort will use all of its political power to mitigate or even reverse the effects. Surely one cannot expect such a politically powerful cohort to simply wait for the natural economic law to decide its financial fate.

Using the results that stock market (through the behavior of Tobin's  $q$ ) will respond positively to immigration flow, the chapter contributes to the two important lines of research with an insight on how both stock market meltdown and the fragility of pension system can be mitigated by allowing more immigration. In other words, I provide another channel through which the native-born citizens can benefit from immigration. Even though the wage earners may suffer from depressing wage in the present period, a life-time consideration opens up a possible future benefit of bringing in immigration to shore up population growth.

The next section outlines the model under deterministic environment. I show within this section that the asset prices will rise with immigration. The section ends with an analysis of the political equilibrium. In section 3, I augment the model with uncertainty and repeat the analysis of both economic and political economic equilibrium. I provide some light empirical thoughts in section 4, citing existing empirical works and paving the way for future empirical works. I conclude in section 5.

## 2.2 Deterministic Model

Consider a 2-period-lived overlapping-generations model in the spirit of Diamond [1965]. On the production side, I follow the work of Abel [2003]. I also augment the model with the demographic structures from Sand and Razin [2007]. I start by describing the technology and its structures. Then later I move on to the details of consumers and demography.

### 2.2.1 Technologies

There is a single consumption good in this economy, produced with two factors: capital ( $K$ ) and labor ( $L$ ). The production technology of the consumption good is assumed to take Cobb-Douglas form

$$Y_t = K_t^\alpha L_t^{1-\alpha}, \quad (2.1)$$

where  $\alpha \in (0, 1)$ .

In addition, the economy also produces a capital good to be used in production



of the consumption good. This capital adjustment technology converts investment and the current stock of capital into capital stock in the next period. The capital production technology is also of Cobb-Douglas form, similar to Abel [2003] and Basu [1987]. Algebraically, I adopt the following specification

$$K_{t+1} = \left( \frac{I_t}{K_t} \right)^\phi K_t \quad (2.2)$$

where  $0 \leq \phi \leq 1$ ,  $K_t$  is the aggregate capital stock and  $I_t$  is the gross investment in the economy in period  $t$ . Note that, under this specification of capital adjustment technology, I implicitly assume *full* depreciation of capital from last period. This capital technology is consistent with equation (4b) in Hayashi [1982]. Because of its concavity with respect to  $I_t$ , I have a cost to capital adjustment.<sup>1</sup>

Letting  $R_t$ ,  $w_t$ , and  $q_t$  denote the rate of return, the wage in period  $t$ , and the shadow price of capital carried into period  $t + 1$  (in terms of consumption good), respectively, I have the following profit-maximizing conditions for the firm with respect to  $K_t$ ,  $L_t$ , and  $I_t$

$$R_t = \frac{\alpha \left( \frac{L_t}{K_t} \right)^{1-\alpha} + q_t(1-\phi) \left( \frac{I_t}{K_t} \right)^\phi}{q_{t-1}}, \quad (2.3)$$

$$w_t = (1-\alpha) \left( \frac{K_t}{L_t} \right)^\alpha, \quad (2.4)$$

$$q_t = \frac{1}{\phi} \left( \frac{I_t}{K_t} \right)^{1-\phi}. \quad (2.5)$$

The first and second equation are a familiar investment Euler and wage equation. The third equation relates the shadow price of capital to the marginal benefit of investment in capital production. There are a few simplifications I should do before moving on. First, I substitute equation (2.5) for  $q_t$  in equation (2.3) to

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<sup>1</sup>There is an alternative interpretation of this model in terms of two-sector production economy. For details, please refer to Abel [2003].

obtain

$$q_{t-1}R_t = \alpha \left( \frac{L_t}{K_t} \right)^{1-\alpha} + \frac{1-\phi}{\phi} \frac{I_t}{K_t}. \quad (2.6)$$

The condition says that the return on a unit of capital in terms of consumption good must equal to the marginal product of capital plus the marginal reduction in adjustment cost next period. If I now use equation (2.2) and (2.5) to find the market value of the capital carried into period  $t + 1$ , I get

$$q_t K_{t+1} = \frac{1}{\phi} I_t. \quad (2.7)$$

Note that, from equation (2.7), to get the desired valued of capital in the next period, I must invest *less* than that value (investment is measured in terms of consumption good and  $\phi \in [0, 1]$ ). Recall that capital fully depreciates every period, thus the result comes from the fact that investment in this period contributes positively to both capital formation ( $K_{t+1}$ ) and to the appraisal value of capital ( $q_t$ ). If  $\phi = 1$ , I have no adjustment cost and investment equals to the value of desired capital ( $q_t = 1$ ).

## 2.2.2 Populations

Following Sand and Razin [2007], I assume that immigrants are naturalized in one period thus gaining citizenship in the next period. All immigrants are young. I let  $\mu_t \in [0, 1]$  denote the inflow of migrants as a proportion of the current native-born young population. I let  $N_t$  denote the size of native young population in period  $t$ , thus the number of immigrants is  $\mu_t N_t$ . Native-born has a population growth rate of  $n \in [-1, 1]$  while the immigrant's growth rate is  $m \in [-1, 1]$ . I assume  $n < m$ , so the immigrants' entrance to the economy will lift national population growth rate. According to these descriptions, the demographic dynamics is described by

the following equation

$$\begin{aligned} N_t &= (1+n)N_{t-1} + \mu_{t-1}(1+m)N_{t-1} \\ &= n_{t-1}N_{t-1} \end{aligned} \tag{2.8}$$

where  $n_{t-1} = 1+n+\mu_{t-1}(1+m)$  be the gross population growth rate at time  $t-1$ .

I assume that both native and immigrants supply their unit of labor inelastically. However, as for their contribution to the labor force, immigrants are a perfect substitute for native-born workers with lower productivity. I capture this differential productivity between the two types of labor in a linear fashion with a parameter  $\gamma \in (0, 1]$ .<sup>2</sup> Given this setup, the aggregate labor supply is given by

$$L_t^s = (1 + \gamma\mu_t)N_t, \tag{2.9}$$

which follows from the assumption that every and only young individual works, including immigrants. Note that,  $(1 + \mu_t)N_t$  is the size of the young consumer in the period  $t$ , while  $L_t^s$  is the size of the labor force. For brevity of notation, I let

$$\gamma^i = \begin{cases} 1 & , i = N \\ \gamma & , i = M \end{cases}$$

where  $i$  indexes either native-born ( $N$ ) or immigrant ( $M$ ) status. It is straightforward to see that the wage receives for each  $i$  is

$$w_t^i = \gamma^i w_t \tag{2.10}$$

where  $w_t$  is given in equation (2.4). Furthermore, the children of the immigrants will be part of the citizens, and hence their labor will be as productive as those of the native born. This is implicitly embedded in the structure of population dynamics in equation (2.8). So only the newly arrived immigrants have lower productivity of labor than that of the native-born labor.

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<sup>2</sup>Arguably, imperfect substitution could be also be considered via a production function with skill complementarity. However, to keep the model tractable, I refrain from doing so. I refer interested readers to the work by Casarico and Devillanova [2003].

### 2.2.3 Preferences

Immigrants and native-born citizens have identical preference. Each individual lives for two periods in an overlapping manner. Individuals consume consumption good in both periods, while supply inelastically his unit labor when young. Therefore, in retirement year, old individual will only consume his saving. I assume no bequest motives between generations.<sup>3</sup> Each individual in period  $t$  has the following logarithmic utility function,

$$\begin{aligned} U_t^o(c_{t-1}^y, c_t^o) &= \ln(c_t^o) \\ U_t^y(c_t^y, c_{t+1}^o) &= \ln(c_t^y) + \beta \ln(c_{t+1}^o). \end{aligned} \quad (2.11)$$

Individuals from each generation will maximize their respective utility given in equation (2.11) subject to budget constraints

$$\begin{aligned} c_t^y + s_t &= w_t^i \\ c_{t+1}^o &= R_{t+1} s_t^i. \end{aligned}$$

Given this log-linear preference, for both  $i \in \{N, M\}$ , I find that  $c_t^{y,i} = \frac{w_t^i}{1+\beta}$  and  $c_{t+1}^{o,i} = \frac{\beta R_{t+1}}{1+\beta} w_t^i$ . Saving of the young is  $s_t^i = \frac{\beta}{1+\beta} w_t^i$ . Only the young saves in the economy, while the old dissaves his savings for old period's consumption. I substitute these expressions back into the utility functions in (2.11) to find each generation's indirect utility function in period  $t$  as

$$\begin{aligned} V_t^{o,i}(w_{t-1}, R_t) &= \ln(w_{t-1}^i) + \ln(R_t) + \ln\left(\frac{\beta}{1+\beta}\right) \\ V_t^{y,i}(w_t, R_{t+1}) &= (1+\beta) \ln(w_t^i) + \beta \ln(R_{t+1}) + \ln\left(\frac{\beta^\beta}{(1+\beta)^{1+\beta}}\right). \end{aligned} \quad (2.12)$$

I want to point out here that savings of immigrants are smaller than that of the native-born by the fraction  $\gamma$ . However, if there is a probability of returning home,

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<sup>3</sup>Abel [2001] shows that bequest will not change the theoretical conclusion of stock market meltdown. So I disregard bequest motives to focus directly at the core issue of interest.

the result could be different, as demonstrated in Galor and Stark [1990]. They show that, with homogeneous labor force, positive probability of returning home will induce immigrants to save more. I do not have that feature in the model.

## 2.2.4 Economic Equilibrium

In equilibrium, the savings of the young finance the capital for the next period:

$$S_t = q_t K_{t+1}$$

where  $S_t$  is the aggregate saving of the young in the economy (including migrants). Note that the next period's level of capital is fully determined in this period when the saving decision is made. I can divide the aggregate saving of the young into two parts: one from the native ( $N$ ) and the other from the migrant ( $M$ ),

$$S_t = N_t s_t^N + \mu_t N_t s_t^M = (1 + \gamma \mu_t) N_t s_t^N$$

where the latter equality follows from using equation (2.10) in the saving function. Hence, the equilibrium condition above can be rewritten as

$$(1 + \gamma \mu_t) N_t s_t^N = q_t K_{t+1}. \quad (2.13)$$

It will be convenient to focus on per native-worker variables such as  $y_t = \frac{Y_t}{N_t}$  and  $k_t = \frac{K_t}{N_t}$ , instead of their aggregate counterparts.<sup>4</sup> Labor market clearing implies that wage per unit of worker is  $w_t = (1 - \alpha) \left( \frac{k_t}{1 + \gamma \mu_t} \right)^\alpha$ . Using equation (2.7), which relates value of capital to investment, and writing condition (2.13) in per native

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<sup>4</sup>Defining  $k_{t+1} = \frac{K_{t+1}}{N_{t+1}}$  has slight peculiarity. At the end of period  $t$ , capital per investor (young of period  $t$ ) is in fact  $\frac{K_{t+1}}{N_t}$ , though at the dawn light of period  $t + 1$ , capital per native worker is  $k_{t+1}$ . The choice then boils down to the importance of each ratio in the analysis. I decide to stick with the latter rather than the first in contrast to Abel [2003].

capita, I have

$$\frac{I_t}{N_t} = \frac{\phi\beta(1-\alpha)(1+\gamma\mu_t)^{1-\alpha}k_t^\alpha}{1+\beta}. \quad (2.14)$$

I can now use this equation to solve the model for all of the variables. First, I use this equation to rewrite the shadow price of capital in equation (2.5) as

$$q_t = \frac{1}{\phi^\phi} \left( \frac{\beta(1-\alpha)}{1+\beta} \right)^{1-\phi} \left( \frac{1+\gamma\mu_t}{k_t} \right)^{(1-\phi)(1-\alpha)}. \quad (2.15)$$

Next, I use of the transformed equation of the rate of return for capital in (2.6) and again substitute in investment per native citizen (2.14) and the labor supply in equation (2.9), and rearrange to have

$$R_t = \frac{1}{q_{t-1}} \left[ \alpha + \frac{\beta(1-\phi)(1-\alpha)}{1+\beta} \right] \left( \frac{1+\gamma\mu_t}{k_t} \right)^{1-\alpha}. \quad (2.16)$$

By observation of equation (2.15) and (2.16), I can summarize impact of immigration on the asset price and rate of return in the following proposition.

**Proposition 8.** *For  $\phi \in (0, 1)$  and  $\gamma > 0$ , increasing the rate of migration,  $\mu_t$ , will increase the price of capital,  $q_t$ , and rate of return on capital,  $R_t$ .*

For the shadow price of capital, the positive effect of immigration feeds through investment-saving channel. Higher immigration leads to higher supply of loanable funds as well as higher demand for it. These two forces increase equilibrium level of investment and push up the price of capital. As for the return on savings, a rise in the rate of migration will induce higher returns on the capital for the holder of already-existing capital. Immigration provides benefits through two channels. First channel is to increase the marginal product of capital in the production of consumption good by simply augmenting the labor force. The second channel is to increase investment in the economy to form a higher capital stock in the next period. Higher investment uplifts  $q_t$  directly as well as further increases the marginal product of  $K_t$  in the *capital adjustment* technology. Together, they allow

the capital holders in the economy to enjoy higher return on their investment. However, this increase of the price one must pay to acquire capital in the same period produces conflicting interest for people who are about to save at time  $t$ . In this period, they would prefer no immigration so the price of capital in period  $t$  would be low, hence they could accumulate more assets. Then as period  $t + 1$  comes, they would prefer as large as possible immigration to drive up the return of their savings.

I pause here a bit to comment on the stock market meltdown literature as a consequent of demographic shift. In this economy, labor flows across border will dampen any effect of the stock market doomsday due to the aging demography. In addition to the expansion of the tax base to sustain the current PAYGO pension system, this channel provides an additional benefit of immigration to the retirees.

Returning to the analysis, I substitute equation (2.14) into the capital technology in equation (2.2), I get

$$k_{t+1} = \frac{1}{n_t} \left( \frac{\phi\beta(1-\alpha)}{1+\beta} \right)^\phi (1 + \gamma\mu_t)^{(1-\alpha)\phi} (k_t)^{(1-\phi)(1-\alpha)}. \quad (2.17)$$

Here the direction of response of  $k_{t+1}$  to the policy variable  $\mu_t$  is unclear. On the one hand, under  $\gamma > 0$ , immigrants always bring some positive contribution to the production through saving and investment. So higher rate of immigration will consequently lead to higher capital going in the next period. On the other hand, the actual capital per capita in the next period depends on the size of population in that period. Depending on how fast immigrants procreate, this effect will adversely affect the level of capital per labor in period  $t + 1$  (recall that gross population growth is denoted by  $n_t = 1 + n + \mu_t(1 + m)$ ).

Since I have assumed log additive preferences, the convenient form of endogenous variables given in equation (2.15) to (2.17) is in log form. Therefore, I take

log of these equations, and utilize the population dynamics from equation (2.8) to write  $K_{t+1}$  in per native form, and I have the following expressions

$$\ln q_t = (1 - \alpha)(1 - \phi)[\ln(1 + \gamma\mu_t) - \ln k_t] + (1 - \phi) \ln \left( \frac{\beta(1 - \alpha)}{1 + \beta} \right) - \phi \ln \phi \quad (2.18)$$

$$\ln R_t = (1 - \alpha)[\ln(1 + \gamma\mu_t) - \ln k_t] - \ln q_{t-1} + \ln \left( \alpha + \frac{\beta(1 - \phi)(1 - \alpha)}{1 + \beta} \right) \quad (2.19)$$

$$\ln k_{t+1} = (1 - \phi(1 - \alpha)) \ln k_t + (1 - \alpha)\phi \ln(1 + \gamma\mu_t) - \ln n_t + \phi \ln \left( \frac{\beta\phi(1 - \alpha)}{1 + \beta} \right). \quad (2.20)$$

In addition to  $R_t$ , I will need to know the behavior of  $R_{t+1}$  in response to today's variables. So I take equation (2.16) and forward it one period. Then I take log of the resulting equation, and substitute in the expressions for  $\ln q_t$  and  $\ln k_{t+1}$  to get

$$\begin{aligned} \ln R_{t+1} = & (1 - \alpha)[\ln(1 + \gamma\mu_{t+1}) - \alpha\phi \ln k_t - (1 - \alpha\phi) \ln(1 + \gamma\mu_t) + \ln n_t] \quad (2.21) \\ & + \ln \left( \alpha + \frac{\beta(1 - \phi)(1 - \alpha)}{1 + \beta} \right) + \phi \ln \left( \frac{\beta}{1 + \beta} \right) + (1 - \phi) \ln \left( \frac{\beta(1 - \alpha)}{1 + \beta} \right) \end{aligned}$$

As in Proposition 8,  $R_{t+1}$  positively respond to change in  $\mu_{t+1}$ . As for the response of  $R_{t+1}$  to  $\mu_t$ , the result seems a little ambiguous. However, under careful analysis, the equation reveals that  $R_{t+1}$  responds positively to  $\mu_t$ . The result follows from the parameter restrictions within setup of the model (both production and demographic parameters).

**Corollary 9.**  $R_{t+1}$  is increasing in period  $t$  choice of immigration quotas,  $\mu_t$ .

I delegate the proof to the appendix. This corollary shows that returns on investment will *continue* to move beneficially as a response *past* level of immigration. Basic intuition dictates that more immigrants would increase saving and hence would further depress the return on savings earned in the next period (by pushing up the price of capital this period relative to the next). However, after each population grows according to their rates, higher population growth rate of immigrants will generate more workers and more demand for assets (also more saving and investment). This raises the return on saving in the next period.



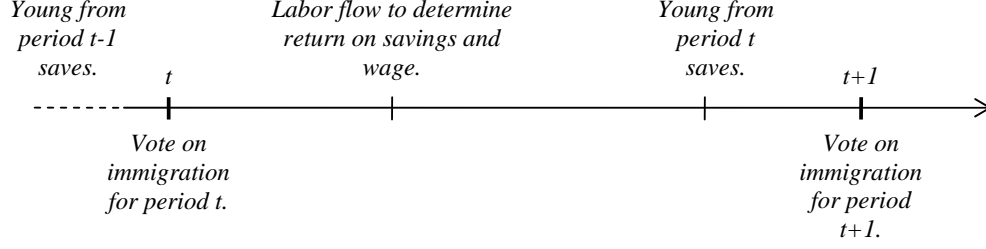


Figure 2.5: The timing in the deterministic model.

### 2.2.5 Political Equilibrium

In this section, I study the nature of political equilibrium in this economy. The timing in the economy is given in Figure 2.5. Voters vote at the beginning of the period, winning policies are implemented, and payoffs are realized. The political process repeats.

The first step in the analysis is to derive completely the indirect utility functions for the contemporaneous generations as a function of policy and state variables. Using the already convenient form of endogeneous variables given in equation (2.18) to (2.21) and the raw indirect utility functions derived in equation (2.12), I have

$$V_t^o(\mu_t, k_t) = (1 - \alpha) \ln \left( \frac{1 + \gamma \mu_t}{k_t} \right) + \ln w_{t-1} - \ln q_{t-1} + B^o \quad (2.22)$$

$$\begin{aligned} V_t^y(\mu_t, \mu_{t+1}, k_t) &= (1 + \beta(1 - \phi(1 - \alpha)))\alpha \ln k_t + \beta(1 - \alpha) [\ln n_t + \ln(1 + \gamma \mu_{t+1})] \\ &\quad - ((1 + \beta)\alpha + \beta(1 - \alpha)(1 - \alpha\phi)) \ln(1 + \gamma \mu_t) + B^y \end{aligned} \quad (2.23)$$

where  $B^j$  for  $j \in \{y, o\}$  are constant for individual's indirect utility. These constants will not play a role in my subsequent analyses. They are

$$\begin{aligned} B^o &= \ln \left( \gamma^i \left[ \frac{\beta}{1 + \beta} \right] \left[ \alpha + \frac{\beta(1 - \phi)}{1 + \beta} \right] \right) \\ B^y &= \ln \left( \beta^\beta \left[ \frac{\gamma^i}{(1 + \beta)} \right]^{1 + \beta} \right) + \ln \left( \alpha + \frac{\beta(1 - \phi)}{1 + \beta} \right) + \ln \left( \frac{\beta}{1 + \beta} \right). \end{aligned}$$

The political economic equilibrium uses the subgame-perfect Markov concept (also known as Markov-perfect). The following definition defines this concept formally in the context of the model.

**Definition 10.** *The policy function  $\mu_t = M(\mu_{t-1}, k_t)$  where  $M : [0, 1] \times \mathbb{R}_+ \rightarrow [0, 1]$  is the decision rule for migration level, constitutes a Subgame Perfect Markov Political Equilibrium (SPME) if it satisfies*

$$\begin{aligned} M(\mu_{t-1}, k_t) &= \arg \max_{\mu_t \in [0, 1]} V_t^d(\mu_t, M(\mu_t, k_{t+1}), k_t) \\ &\text{subject to } k_{t+1} = \frac{1}{n_t} \left( \frac{\phi\beta}{1+\beta} \right)^\phi (1 + \gamma\mu_t)^{(1-\alpha)\phi} (k_t)^{(1-\phi)(1-\alpha)} \end{aligned}$$

where  $n_t = 1 + n + \mu_t(1 + m)$  and  $d \in \{y, o\}$  is the identity of the decisive voter.

For a cleaner notation, I will denote the evolution of capital per native worker with  $k_{t+1} = K(\mu_t, k_t)$ . Since the indirect utility functions in equation (2.22) - (2.23) have already incorporated in the constraint of  $k_{t+1}$ , I can simply continue with characterizing the SPME with the derived equations. Drawing attention now to the definition, I note that it requires the SPME to be optimal given that the decisive voter takes into account that the policy decision rule will be applied in the next period.

The ratio of old versus young voter in period  $t + 1$  is given by the following expression (equation (13) in Sand and Razin [2007])

$$v_{t+1} = v(\mu_t) = \frac{1 + \mu_t}{1 + n + \mu_t(1 + m)}. \quad (2.24)$$

In this simplistic setup, I think of this ratio,  $v(\mu_t)$ , as the old dependency ratio of the economy. This ratio, along with relative magnitude of the underlying demographic parameters, will help determine the identity of the decisive voter in the next period as a function of  $\mu_t$ . I assume the tie breaker goes to the old voter, that

is, if  $v_{t+1} \geq 1$ , the old is the decisive median voter. Recall that I always assume  $n < m$ . Accordingly, if both  $m, n > 0$ , then  $v_{s+1} < 1$  for all  $\mu_s \in [0, 1]$  and for all  $s \geq t$ . On the other hand if  $m, n < 0$  then  $v_{s+1} > 1$  for all  $\mu_s \in [0, 1]$  and for all  $s \geq t$ . An interesting case is when  $n < 0 < m$ , which has a potential of producing a switching demographic structures. Now I look at the political equilibrium.

**Proposition 11.** *The policy decision rule that constitutes a Subgame-perfect Markov Political Equilibrium is described by*

$$M(\mu_{t-1}, k_t) = \begin{cases} 1 & , \text{ if } v(\mu_{t-1}) \geq 1 \\ \mu^* & , \text{ if } v(\mu_{t-1}) < 1, \gamma < \underline{\gamma}, \text{ and } \Psi(\mu^*) \leq 1 \\ \min \left\{ \mu^*, -\frac{n}{m} \right\} & , \text{ if } v(\mu_{t-1}) < 1, \gamma < \underline{\gamma}, \text{ and } \Psi(\mu^*) > 1 \\ 0 & , \text{ if } v(\mu_{t-1}) < 1, \gamma \geq \underline{\gamma} \end{cases} \quad (2.25)$$

where the parameters  $\mu^*$ ,  $\underline{\gamma}$  and  $\Psi(\mu)$  are defined in the appendix and capital evolves according to the strategy specified above,

$$k_{t+1} = K(k_t, M(\mu_{t-1}, k_t)).$$

Again I delegate the proof to appendix. The evolution of the capital is simply the equation in Definition 10 with policy rule substituted in for  $\mu_t$  (depicted in Figure 2.6). Similar to the line of literatures using SPME with overlapping-generations model (for example, see Hassler [2003]), I have the equilibrium with the behavior that the current period's decisive voter tries to influence the identity of the next period's decisive voter. This result reflects the preference of the young individuals. Sure, they would like to choose their optimum level of migration rate (that is  $\max \{0, \mu^*\}$ ), where  $\mu^*$  comes from maximizing the young's indirect utility. But if such a choice will take away the privilege of being the decisive voter in the next period, they may compromise for a different level of migration rate (as close

as possible to their preferred policy choice) that will guarantee themselves a say in the next period. Such a strategic behavior will arise or not depends on the relative distance from  $\mu^*$  to 1, and from  $\mu^*$  to  $-\frac{n}{m}$ . If  $\mu^*$  is close enough to 1, the young in this period will choose his optimal  $\mu^*$ , knowing that next period's median voter will be young and they will also choose  $\mu^*$ . Since  $\mu^*$  is relatively closer to 1 (the current period's young's preferred level of next period's rate of immigration), the current young will not be hurt as much by giving up the decisive role to the next generation while getting to determine the policy at their ideal point in the current period. However, if  $\mu^*$  is close to  $-\frac{n}{m}$ , in the viewpoint of the young generation, it is worth sacrificing a little today (by choosing  $-\frac{n}{m}$  instead of  $\mu^*$ ) to gain a lot tomorrow (by placing their generation as the decisive voter in the next period). The function  $\Psi(\mu)$  captures these relative tradeoffs.

The parameter  $\underline{\gamma}$  tells me the threshold where some positive immigration is desirable. Above the threshold, the young in this period are better-off forbidding immigration altogether. The result suggests that for the young generation of a high productivity country, low-skilled immigrant may make them better-off due to less competition.

**Corollary 12.** *Depending on the parameters of population growth rates, there are three possible equilibrium paths:*

1. *For  $0 < n$ , the young individuals form the majority in all periods and the immigration rate will be  $\max\{0, \mu^*\} < 1$ .*
2. *For  $n + m < 0$ , the old individuals form form majority in all periods and the immigration rate is always 1.*
3. *For  $n < 0 < n + m$ , the old and young individuals take turn to form the majority in the economy. Consequently, there would be less than a unit mi-*

*gration rate when young individuals form the majority and a unit migration rate when old individuals form the majority. The economy cycle through different immigration policies.*

It is quite natural to expect three possible equilibrium paths to emerge, given the description of the demographic structure before Proposition 11. When  $0 < n$ , the young generation always forms the majority. Depending on the parameter values, the young will either prefer 0 or the level of migration equals to  $\mu^* < 1$ . There is no game to play here. Similarly, for  $n + m < 0$ , the old generation forms the majority, hence they always open the economy to migration, trying to boost the return on their savings. Consequently, the immigration rate equals to one. Again, there is no demographic game here, so the result is straightforward. When  $n < 0 < n + m$ , strategic behavior of the young causes switching between old and young as the decisive voter. Above SPME applies directly to this case when there is a possible game among the two co-existing generations.

I end this section with a corollary linking the political equilibrium in Proposition 11 to the behavior of asset price in Proposition 8, see accompanying Figure 2.6.

**Corollary 13.** *The asset price in period  $t$ ,  $q_t$ , and the return on saving in period  $t$ ,  $R_t$ , will be higher when the old cohort forms the majority than when young forms the majority.*

This result requires no proof as it is fairly self-evident. Upon inspection,  $q_t$  rises with immigration quota, which will be higher when old cohort has the majority than when young has the majority.

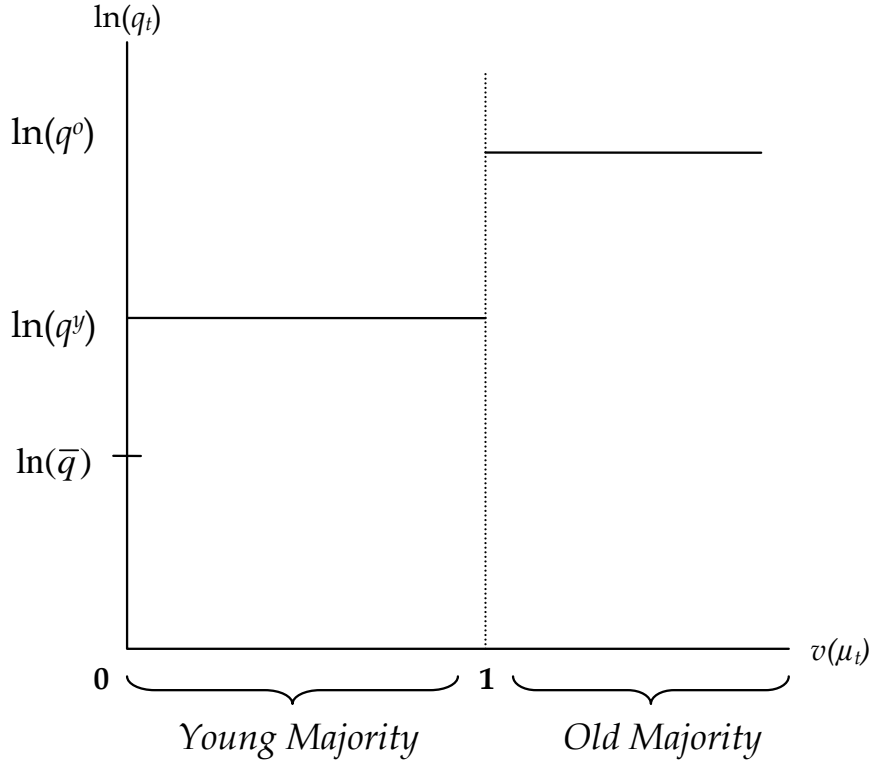


Figure 2.6:  $\ln(\bar{q})$ ,  $\ln(q^y)$ , and  $\ln(q^o)$  is the natural log level of asset prices in when there is no immigration, the young forms majority, and the old forms majority, respectively.

### 2.3 Economy under Uncertainty

Often, the consideration of factor flow is a balance with between optimal risk and return management. My aim now is capture this with some alterations in the model, allowing some uncertainty in the model. Namely, I use a random immigrant's growth rate to capture the idea that their growth rate cannot be predicted with precision. I also introduce stochastic total factor productivity (TFP) into the production function of the consumption good. Nonetheless, this last addition to the model will not produce significantly different result to having just the uncertain population growth. Thus this section can be viewed as providing a robustness check for my earlier results.

### 2.3.1 Production

To avoid repetition, I will try to restate only the parts of the model with alterations.

Production now takes the form

$$Y_t = A_t F(K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha}$$

where the TFP of the model is a random variable. I will not specify now the stochastic form that  $A_t$  might follow, as such information is not necessary for the result. The capital adjustment technology takes the same form as in equation (2.2), hence it will induce the same price of capital as in equation (2.5). The marginal returns to labor and capital take similar form only with the TFP floating around

$$w_t = (1 - \alpha)A_t \left(\frac{K_t}{L_t}\right)^\alpha \text{ and } R_t = \frac{\alpha A_t \left(\frac{L_t}{K_t}\right)^{1-\alpha} + \frac{1-\phi}{\phi} \frac{I_t}{K_t}}{q_{t-1}} \quad (2.26)$$

where I have already substitute in for  $q_t$  in the expression for  $R_t$ . It will also be useful for future reference to recall equation (2.7), stating that  $q_t K_{t+1} = \frac{1}{\phi} I_t$ .

### 2.3.2 Populations

Other population structures are no different from above, except that I allow for a random population growth rate of immigrants. That is, I assume  $m_t \sim [-1, 1]$  for every  $t \geq 0$  with its mean greater than the population growth rate of the natives (i.e.  $\mathbb{E}_{t-1} m_t \geq n$ ). I also assume they are *independent* over time of any lagged or forward values. I still let  $\mu_t \in [0, 1]$  be the proportion of immigrant to native workers. Hence expected population size in period  $t$  given the immigration level

in period  $t - 1$  is<sup>5</sup>

$$\mathbb{E}_{t-1}N_t = (1 + n + \mu_{t-1}(1 + \mathbb{E}_{t-1}m_{t-1}))N_{t-1}$$

where the symbol  $\mathbb{E}_{t-1}$  denotes expectation conditional on the information set at time  $t - 1$ . As before, all entering immigrants are young, and they supply their unit labor inelastically. Since there is a productivity difference, I continue to denote the two wages with one notation  $w_t^i$  as described before equation (2.10).

### 2.3.3 Preference

Ideally, it would be desirable to have a model that captures the trade off between risks and returns with a parameter like the degree of risk aversion. However, such a model proves difficult to work with for closed-form structures that I want. In order to continue with my analysis, I will have to forego more exotic utility functions and settle for an easier log preference that is more manageable. Both cohorts, young and old, will choose consumption bundle within the budget set that maximizes their expected utilities:

$$EU_t^{y,i}(c_t^{y,i}, c_{t+1}^{o,i}) = \mathbb{E}_t \{ \ln c_t^{y,i} + \beta \ln c_{t+1}^{o,i} \} \text{ and } EU_t^{o,i}(c_t^{o,i}) = \mathbb{E}_t \{ \ln c_t^{o,i} \},$$

where  $i \in \{m, n\}$  denotes individual's immigration status. The budget constraints for consumers look similar to the deterministic case, only now it must hold almost surely. Under log preference, young's decision to save is independent of any future returns it might earn. So the optimal profile of consumption and saving looks like what I have presented before, which I can substitute in the expected utilities above to find relevant expected indirect utilities.

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<sup>5</sup>In line with my description in the deterministic setup, I will also refer to this as "expected population dynamics".



### 2.3.4 Economic Equilibrium

I will briefly describe the steps taken to derive analytical expressions here, as details are similar to the deterministic setup. In equilibrium, the capital going into production next period is financed solely from the savings of the young (since the young cohort is the only saver, while the old cohort only dissaves), so the equilibrium condition is  $S_t = q_t K_{t+1}$ . With the help of equation (2.26), market clearing condition in the labor market ( $L_t = (1 + \gamma\mu_t)N_t$ ), and the population dynamics, I can write aggregate saving as  $S_t = (1 + \gamma\mu_t)N_t(1 - \alpha)\frac{\beta}{1+\beta}A_t\left(\frac{k_t}{1+\gamma\mu_t}\right)^\alpha$ , where  $k_t$  denotes capital per native-born worker. Now I use the relationship that  $q_t K_{t+1} = \frac{1}{\phi}I_t$  to find the expression for aggregate investment. Having found the expression for  $I_t$ , I am now ready to find the important equations for the endogenous variables. These are given as follows,

$$\begin{aligned}
\text{returns on capital:} \quad R_t &= \frac{1}{q_{t-1}} \left[ \alpha + \frac{\beta(1-\phi)(1-\alpha)}{1+\beta} \right] A_t \left( \frac{1+\gamma\mu_t}{k_t} \right)^{1-\alpha} \\
\text{shadow price of capital:} \quad q_t &= \frac{1}{\phi^\phi} A_t^{1-\phi} \left( \frac{\beta(1-\alpha)}{1+\beta} \right)^{1-\phi} \left( \frac{1+\gamma\mu_t}{k_t} \right)^{(1-\phi)(1-\alpha)} \quad (2.27) \\
\text{capital per native in } t+1 : \quad k_{t+1} &= \frac{\phi^\phi A_t^\phi \left( \frac{\beta(1-\alpha)}{1+\beta} \right)^\phi k_t^{1-\phi(1-\alpha)} (1+\gamma\mu_t)^{\phi(1-\alpha)}}{1+n+\mu_t(1+m_t)}.
\end{aligned}$$

I now proceed by taking natural log of these expressions and put them in the expected indirect utility functions to facilitate political economic analysis.

### 2.3.5 Political Equilibrium

When studying the political equilibrium under uncertainty, the timing of the model is more important than ever before. It ultimately decides what information the agents in the economy possess when evaluating their choices. The timeline of the

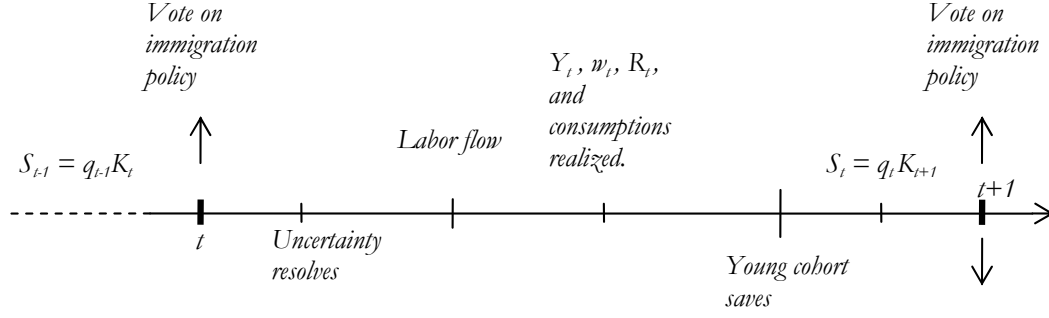


Figure 2.7: The timing of the model with uncertainty.

model I have in mind is illustrated in Figure 2.7. I want the voting in the model to take place in every period before the uncertainty is resolved. Another way of thinking about this is to imagine the young agent born at the dawn of period  $t$ . When political process takes place (possibly right after birth of the youngsters!), both cohorts are merely equipped with information set from period  $t-1$ , as nothing new in period  $t$  has been revealed yet.<sup>6</sup> Thus at the point where political decisions are made, people take expected value of their indirect utilities conditional on information in period  $t-1$ .<sup>7</sup> After the policy is determined, nature reveals the uncertain values, then labor flows, and consumptions are realized.

The expressions for the expected indirect utilities of the representative agent of each cohort, ignoring unimportant constants, are given below:

$$\begin{aligned}
 EV_t^o &= \mathbb{E}_{t-1} \left\{ (1-\alpha) \ln \left( \frac{1+\gamma\mu_t}{k_t} \right) + \ln A_t \right\} + \ln \{w_{t-1}\} - \ln q_{t-1} \\
 EV_t^y &= \mathbb{E}_{t-1} \left\{ +\beta(1-\alpha) \left[ \ln(1+\gamma\mu_{t+1}) + \ln \left( \frac{1+n+\mu_t(1+m_t)}{k_t} \right) \right] + \beta \ln A_{t+1} \right. \\
 &\quad \left. + (1+\alpha\beta\phi) \ln A_t + [\beta + \alpha(1-(1-\alpha)\beta\phi)] \left[ \ln \left( \frac{k_t}{1+\gamma\mu_t} \right) \right] \right\}.
 \end{aligned}$$

<sup>6</sup>Doing so, I stay inline with the literature where information at time  $t$  contains information about all variables in time  $t$ .

<sup>7</sup>I can do this in two ways. One by brute assumption that agents take expected indirect utility conditional on information set time  $t-1$  during political process,  $\mathbb{E}_{t-1}(\cdot)$ . Another is to assume that agents take expected value of expected indirect utility of period  $t$ , or  $\mathbb{E}_{t-1}(\mathbb{E}_t(\cdot))$ , which leads to the same conclusion.

Note that no cohort needs a superscript  $i \in \{m, n\}$ , as only the young native at the beginning of the period gets to vote, and the part of the preference that matters for the old matches for both native and previously-migrated old.

I want to find the ideal immigration policy of the young as I have done in the proof of Proposition 11. Differentiating the young's expected indirect utility with respect to  $\mu_t$  holding future variables constant yields the first order condition

$$\mathbb{E}_{t-1} \left\{ \frac{(1 + \gamma\hat{\mu})(1 + m)}{1 + n + \hat{\mu}(1 + m)} \right\} = \frac{\gamma(\beta + \alpha[1 - (1 - \alpha)\beta\phi])}{\beta(1 - \alpha)}, \quad (2.28)$$

which implicitly defines the immigration bliss point of the young cohort,  $\hat{\mu}$ . I can learn a little bit more about  $\hat{\mu}$  with a help of Jensen's inequality. I define the following quantity: let  $\mu^*$  be the risk-neutral value to the preferred policy of the young. With a little abuse of notation,  $\mu^*$  also represents the ideal immigration policy for the young cohort under deterministic set up, if  $\mathbb{E}_{t-1}m_t = m, \forall t \geq 0$ , where  $m$  is the deterministic population growth rate of the immigrants. I have the following lemma.

**Lemma 14.** *The young prefers lower level of immigration when there is uncertainty about its future growth rate,  $\hat{\mu} \leq \mu^*$ .*

Proof is provided in the appendix. The lemma is actually quite straightforward if one knows a little bit of choice theory under uncertainty. By admitting immigrants, the young generates uncertain population growth rate against themselves. Being a risk-averse individual, this lowers his payoff. So there is a premium associated with his preferred policy level.

**Beliefs.** I will use the same equilibrium concept as defined above in the deterministic setup (see Definition 10). However, before I can actually characterize the equilibrium, there is one issue that I must deal with. It is clear that, irrespective

of the uncertainty, the old cohort always prefers maximal openness of the economy to migration. As for the young, the preference is no longer that clear, because he still has the option of acting strategically. Since the young cannot be sure of his influence of the identity of the decisive voter next period (due to uncertain population growth rate), his expectation (or belief) about the identity of future decisive voter plays a role. The young's belief must be rational, in a sense that it conforms with the logic and parameters of the model. For example, it is implausible for the young to believe that  $\mu_t = 1$  will still induce themselves as the majority old in the next period, or  $\mu_t = 0$  will place the identity of future median voter with the future young (recall that strategic interaction only happens with  $n < 0$ ).

Formally, the young in period  $t$  will expect themselves to be the majority old in period  $t + 1$  if

$$\begin{aligned} (1 + \mu_t)N_t &\geq (1 + n + \mu_t(1 + \mathbb{E}_{t-1}m_t)N_t, \text{ or} \\ -n &\geq \mu_t\mathbb{E}_{t-1}m_t \end{aligned}$$

and if  $\mathbb{E}_{t-1}m_t > 0 > n$ , it leads the condition that  $\frac{-n}{\mathbb{E}_{t-1}m_t} \geq \mu_t$ . I follow Hassler et. al. [2003] in restricting the belief to be monotonic. So the belief of the young can be summarized by a parameter  $\zeta \in \left[0, \frac{-n}{\mathbb{E}_{t-1}m_t}\right]$ . That is, the choice of any  $\mu_t \leq \zeta$  will induce the young to believe that they will be the future majority when they are old,  $v(\mu_t) \geq 1$ , and choose  $\mu_{t+1} = 1$ .

With all the pieces together, I am ready to state the main proposition characterizing the political economic equilibrium in the model with uncertainty.

**Proposition 15.** *For any threshold belief parameter  $\zeta \in \left[0, -\frac{n}{\mathbb{E}_{t-1}m_t}\right]$ , there is a*

*Subgame-perfect Markov Political Equilibrium with the following policy rule*

$$M(\mu_{t-1}, k_t) = \begin{cases} 1 & , \text{ if } v(\mu_{t-1}) \geq 1 \\ \hat{\mu} & , \text{ if } v(\mu_{t-1}) < 1 , \text{ and } \hat{\Psi}(\hat{\mu}, \zeta) \leq 1 \\ \max \{0, \min \{\hat{\mu}, \zeta\}\} & , \text{ if } v(\mu_{t-1}) < 1 , \text{ and } \hat{\Psi}(\hat{\mu}, \zeta) > 1 \end{cases}$$

where the parameters  $\hat{\mu}$  is defined from equation (2.28) and the function  $\hat{\Psi}(\hat{\mu}, \zeta)$  is defined in the appendix. Capital evolves according to the strategy specified above in equation (2.27),

$$k_{t+1} = K(k_t, M(\mu_{t-1}, k_t)).$$

The proof is provided in the appendix. The proposition looks similar to the one presented before in the deterministic case. Depending on the relative distance between the young's belief and the young's bliss point, if the decisive voter represents the young generation, he may have an incentive to act strategically. That is by choosing a suboptimal level of migration to his preference today, in order to make sure that he could pick the policy he wants tomorrow. Note that, the only uncertainty that matters for my result here is the uncertain population growth of the immigrants. It makes the identity of the decisive voter in the future period uncertain, and hence reduces the preferred level of immigration for the young cohort.

Let me assume a little further about the stochastic nature of this model. Let the total factor productivity (TFP) evolves according to a geometric random walk process. That is,

$$\ln A_t = \ln A_{t-1} + \varepsilon_{A,t}.$$

Under this process, I can derive the following stochastic processes for the underly-

ing state variables in the model

$$\begin{aligned} \ln \widehat{k}_{t+1} &= \phi \ln \left( \frac{\phi \alpha \beta}{1 + \beta} \right) + [1 - (1 - \alpha)\phi] \ln \widehat{k}_t + \phi(1 - \alpha) \ln(1 + \gamma \mu_t) \\ &\quad - \ln(1 + n + \mu_t(1 + m_t)) - \frac{1}{1 - \alpha} \varepsilon_{A,t+1} \end{aligned} \quad (2.29)$$

$$\begin{aligned} \ln q_t &= [1 - \phi(1 - \alpha)] \ln q_{t-1} + (1 - \alpha)(1 - \phi) [\ln(1 + \gamma \mu_t) - \ln(1 + \gamma \mu_{t-1})] \\ &\quad + (1 - \phi) [\varepsilon_{A,t} + (1 - \alpha) \ln(1 + n + \mu_{t-1}(1 + m_{t-1}))], \end{aligned} \quad (2.30)$$

where  $\widehat{k}_t = \frac{K_{t+1}}{A_{t+1}^{\frac{1}{1-\alpha}} N_{t+1}}$  is the capital per effective native worker. The following corollary follows readily from equation (2.30) and Proposition 15. Note that I only make additional assumption about the stochastic evolution of TFP for elegant and simplified expressions above. So the validity of the following corollary does not hinge on this assumption at all.

**Corollary 16.** *The asset price,  $q_t$ , will be higher when the old forms majority than when the young forms majority in the economy.*

The proof is evident in equation (2.30). To complete the reasoning, I substitute in  $M(\mu_{t-1}, k_t)$  for  $\mu_t$ . Just like the earlier analysis in the deterministic case, asset prices responds positively to the policy variable. Even with uncertainty, it does not stop the dwellers in the economy to try utilizing the political power for their gains. Although growth rate of immigrant is uncertain, today's choice of migration quotas has a definite and positive impact on the asset valuation. Furthermore, equation (2.30) tells me that the past choice of immigration rate affects the asset prices in two ways: negatively through direct impact and positively through the population growth rate. It can be shown that under the parameter restrictions in the model, the net effect of past immigration rate is a positive one.

**Lemma 17.** *Under geometric random work technological process assumption, if  $1 + m_{t-1} > \gamma(1 + n)$ , then higher immigration in the previous period raises the current period's asset price.*

To see why this is true, differentiate equation (2.30) with respect to  $\mu_{t-1}$ , holding everything else constant. Since it is assumed in the Lemma that  $1 + m_{t-1} > \gamma(1 + n)$  and  $\gamma \in [0, 1]$ , then asset prices will be higher this period with higher  $\mu_{t-1}$ .

## 2.4 Empirical Epilogue

In this section, I briefly review existing empirical evidences available that are related to this chapter. Although some works have been done on immigration impact on returns to capital (Simon [1999]), no works have been done thus far on asset prices. This leaves room for an empirical contribution. In addition, most works on the economic effect immigration pay enormous attention to today's impact of today's level of immigrants. As my intertemporal approach suggests, the impact of immigration today could last for must longer than just their period of entrance. In a pilot study funded by the National Research Council [1997], it projects that if immigration continues as they did in 1995, two third of population growth in the U.S. in 2050 would come from immigrants. And immigration today is actually higher than the level in 1995 (both in absolute number of immigrants and in relative to the U.S. population size). So future impact from the current immigration policy should be of an importance.

In terms of immigrants' impact on the value of assets, Albert Saiz [2007] finds that an influx of immigrants at 1% of a city population is associated with 1% increase in rental and house prices in the city. He argues that this impact is much larger than what the literature found in labor markets. He concludes that homeowners benefit from higher housing prices in net.

The last piece of empirical finding that I would refer to is Brooks [2006]. In that paper, Robin Brooks attempts to find a conclusive evidence regarding demography change and asset prices. While theoretical works conclude that asset prices will likely fall as babyboomers retire, empirical findings reach much less consensus. Brooks uses a comprehensive data set spanning 16 countries, some covering over a century. Instead of relying on a specific measure of demographic ratio, Brooks' empirical specification allows for the entire demographic distribution to enter the regressions. He finds that, in equity-based economy, such as Australia, Canada, New Zealand, the U.K. and the U.S., asset prices respond positively to aging population. That means, as the importance of the old-age group rises, so do the asset prices. Theoretical prediction of falling asset prices as babyboomers retire instead conform with the data from bank-based economies, which raises a question on its validity. Being a bank-based economy, it is unclear whether the demographic variables drive such trends.

All in all, I can say that there is some empirical supports to the findings in this chapter, albeit not directly. Nonetheless, since each period in these theoretical models corresponds to over 20 years (for my model, I would say around 25-30 years), any precise empirical relationship will be difficult to identify as many things happen along those times. For example, stock market in the U.S. booms throughout the 1990's, and so is the rising ratio of babyboomers to the rest of the population. It will be dubious to conclude a causal relationship from babyboomers to stock market, neglecting all other technological advancement during these times. Any future work that aims to link babyboomers and stock market must provide a convincing argument of why the explanation should come from demographic variables and not from others. In addition, it will be important to separate out any endogeneous effect on policy variable from the babyboom cohort. As depicted in



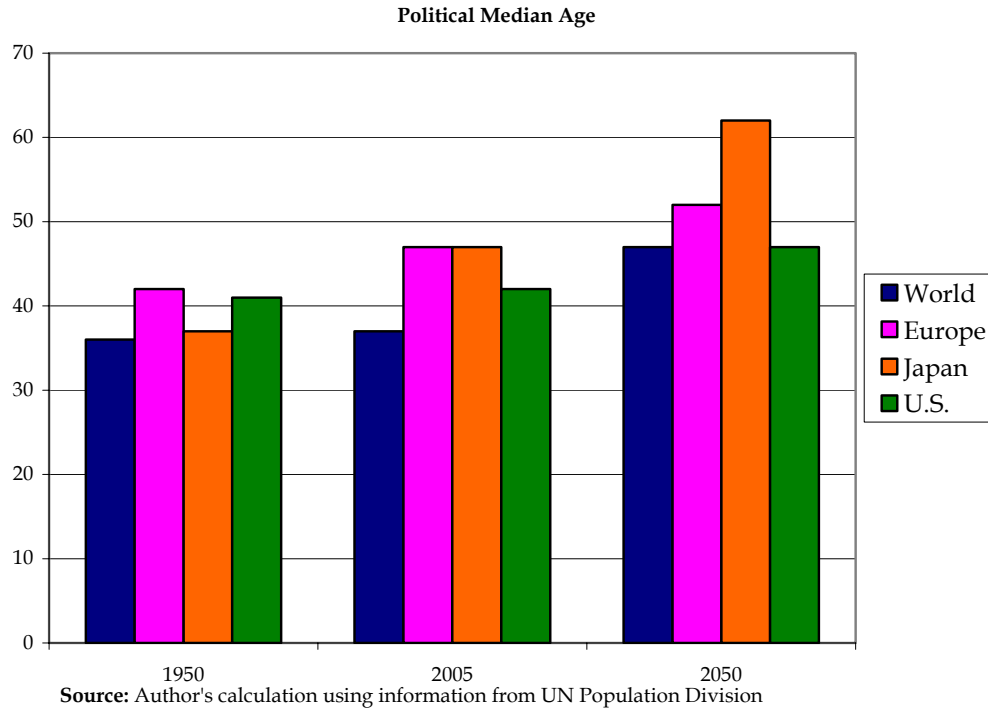


Figure 2.8: Political median (median age of population 18 and over).

Figure 2.8, the age of the median voter is growing through this demographic shift. Hence future analyses must not only focus on an increase in old-age ratio, but also an increase in the political pressure that comes with this transition.

## 2.5 Conclusion

The objectives of my study are twofold. First, I try to understand the relationship between immigration and asset prices. Secondly, I ask how will different cohorts harness the benefits through political interactions. I start with an overlapping-generations model of Diamond's type (Diamond [1965]), which is extended in Abel [2003]. I further include the demographic structure that allows for different population growth rates between native-born citizens and immigrants, and also a political

choice of immigration quota, following Sand and Razin [2007]. In the model, economic factors interact through the production function with an adjustment cost of capital similar to Hayashi [1982] and Basu [1987]. This generates a Tobin's  $q$  that I interpret as reflecting the behavior of asset prices. After looking at the economic equilibrium, I consider political economic equilibrium, using Subgame-perfect Markov Political Economic Equilibrium (SMPE), similarly used and defined in Krusell and Rios-Rull [1996], Hassler et. al. [2003], and Sand and Razin [2007]. The crux of my results are the following.

Asset prices respond positively to immigration in the same period. They also respond positively to the past level of immigration, through higher population growth. By increasing immigration, this pushes for higher investment-saving, thus raising the price of capital by increasing the marginal contribution of each investment to capital adjustment technology. The return to capital could end up much higher than previously thought in Berry and Soligo [1969], and Simon [1999]. So even without any taxes paid, the immigrants' contribution to production, their savings, and the savings and the contributions of their future offsprings can create a favorable movement in asset prices in an economy with a large set of capital holders. With the economy under uncertainty, the core results of my chapter remain fairly robust (only now in expected terms, not absolute). However, uncertainty has a cost to risk-averse individuals. Consequently, the utility-maximizing level of immigration quota is smaller when the population growth rate of the immigrants is uncertain compare to that in the deterministic case.

In all settings, asset prices will be higher when the old cohort forms the majority than when the young cohort forms the majority. In addition, the young generation may have strategic motive while voting (trying to influence the identity

of next period's decisive voter), and the old simply prefers full openness to immigration. However, whether the young acts strategically or not depends, among other things, on relative position of the ideal policy choice and strategic choice. The young cohort will compare costs and benefits between the two schemes and will act according to how the payoffs dictate. The economy may cycle through opening and closing to labor flows because of this strategic behavior.

One important implication from my study shows that asset prices, captured by Tobin's  $q$ , may actually rise in response to aging demography. Since the baby-boomers are a large cohort, they form a strong political clout. Knowing that if they just wait and do nothing, their retirement funds may be jeopardized, baby-boomers will look to outside for buyers and will exert political influence to ensure that their lifetime investment does not plunge in values just when they need it the most. Earlier works in this line ignore completely the powerful political power that lies in the hand of the babyboomers.

In sum, I redirect the attention to a newer channel of analysis. I argue that debates on immigration should extend beyond labor market and fiscal effects. Immigration has potential valuation effect, in terms of asset prices. The future pursuit of this reasoning should dig deeper into empirical works to show how much "potential" there is. Some academic works have already taken those routes, namely Saiz [2003, 2007], and Lach [2007]. I would wish to follow down such a path to throw further lights on the issue.

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## CHAPTER 3

### ARE WE THERE YET? ESCAPING POVERTY TRAP WITH REDISTRIBUTION

#### 3.1 Introduction

A discourse on development can never be complete without mentioning the "vicious cycle of poverty." Development economists have worked tirelessly to uncover many "poverty traps" that create such vicious mechanism on the world's poor. We know now that an emergence of poverty traps require two necessary ingredients: (i) underlying heterogeneity (that is, different initial conditions) and, (ii) a self-reinforcing mechanisms that perpetuate poverty overtime. In particular, as Azariadis and Stachurski [2005] puts it:

"... the mechanisms which reinforce poverty may occur at any scale of social and spatial aggregation, from individuals to families, communities, regions, and countries. Traps can arise not just across geographical location... but also within dispersed collections of individuals affiliated by ethnicity, religious beliefs or clan."

Thus, the existence of poverty traps poses a major obstacle and challenge for all development economists and practitioners. For a comprehensive survey of the literatures on poverty traps, I refer the readers to Azariadis and Stachurski [2005]. Matsuyama [2005] provides a brief, yet insightful introduction to the topic.

Consider a standard poverty trap model with one state variable summarizing each individual's wealth. Conventional wisdom from the literatures tells us that

egalitarian redistribution may end up harming the economy. If the initial conditions are such that the economy's average wealth falls short of the threshold, equal distribution means every member of the economy will be below the threshold, and hence "trapped" in an undesirable equilibrium. Accordingly, the best one-shot redistribution policy for any poor country is to push as many dynasties as possible pass the threshold, which in turn, is equivalent to minimizing the headcount ratio (see Basu [1997] pp. 60-61). This may even entail taxing the poorer quarter and giving the proceeds to the richer households, in order to minimize any transfer of wealth. These findings are rather discouraging and disturbing for poorer economies who started with less than others.

The objective of this chapter is not to unearth a new source of poverty trap previously overlooked by the literatures. I aim to address two simple, but subtle, questions: Can redistribution help all the poor households escape the poverty traps? And if so, will the redistribution policy survives the political process to deliver its promise? Clearly, before any light can be thrown on the second question, I must answer the first question.

As I will show in this chapter that, there is a redistribution scheme that can be used as an escape out of the poverty trap. To elucidate on this point, I focus on a particular model, initially studied in Galor and Zeira [1993]. Although my result on redistribution could straightforwardly be generalized, there are two reasons why I choose to restrict myself in this way. First, with a specific model of poverty traps, I can discuss comparative static exercises and implications using the model's structural parameters. Much more and sharper insights can be drawn from the study that way. The second reason pertains to the second question I want to address. Specific model allows me to write down a closed-form system, in which I



could further investigate the political preference of individuals in the economy.

A theoretical work by Galor and Zeira [1993] is one of the most eminent and seminal accounts of poverty traps. In this influential paper, the authors have demonstrated the importance of initial distribution to macroeconomic outcomes. They focus on how income distribution affect the long-run performance of an economy through investment decisions in human-capital. In the presence of capital market imperfections and indivisible cost to human-capital, the model exhibits multiple steady states, one is preferable to the other. Regrettably, as discussed above, equal distribution does not equal good equilibrium. Non-unique steady state implies that the initial level of wealth will eventually determine who ends up where. The long-run performance of a country is "affected by the initial distribution of wealth, or more specifically by the percentage of individuals who inherit a large enough wealth to enable them to invest in human capital (Galor and Zeira [1993], p. 51)."

Using the framework of Galor and Zeira, I carefully crafted a scheme of taxes and transfers as an escape from the poverty trap. In plain words, my method is as follows. First, I push a dynasty pass the poverty threshold, and then I patiently wait for the wealth accumulation for both the poor and the rich households. Once sufficient amount of wealth is accumulated in the economy, I again push another dynasty over the threshold. By recognizing the usefulness of the economic growth, I repeat the process until every dynasty is above the poverty trap. This result is only possible in dynamic economy which is evolving all the time.

I further show that this redistribution scheme will achieve the result of economic transformation in finite time, not just in its limit. Since there are a finite number

of dynasties within the model<sup>1</sup>, my result guarantees that everyone will be on the path to higher steady-state in finite number of periods. Hence I do not need to appeal to any property that may emerge as the model moves ad infinitum. This provides a support to the idea that gradual transformation can achieve superior economic outcome than the "big-bang" style. However, it should be emphasized that, despite this possibility of an escape out of the poverty trap, poverty tends to be long-lasting<sup>2</sup>.

Once I have these results in hands, I move on to address the issue of its political implementation. Unfortunately, the result for this part is negative. The poor and unskilled dynasties are the main beneficiaries of the policy. As the economy grows richer, the skilled dynasties will collectively form a stronger political bloc, and eventually the redistribution scheme will be stopped. This struggle gives us a glimpse of how inequality may slowdown poverty reduction effort, though not necessarily growth. As an economy transits into the middle income group, it may appear more difficult to reduce poverty at the same rate as before. I discuss implications of this result and also its limitations in the discussion section. I suggest that this result should raise awareness about redistribution and incentives of the skilled workers. The policymakers must not rely entirely on redistribution as the source of income for the poor households.

The question of income distribution and growth occupies the great economic minds for a long time. Arguably, Kuznets [1955] generates the first wave of interest of the profession on this issue. Thereafter, the question breaks into two strains. One line continues to ask the question true to the original inquiry: do growth and

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<sup>1</sup>Original Galor and Zeira [1993]'s formulation uses a continuum of dynasties, while I use finite number of dynasties. This tweaking in the original setup changes nothing of the original work. I could also take a continuum of households and divide them in uniform masses, and proceed with the same analysis. I thank Joseph Zeira for pointing this out to me.

<sup>2</sup>I thank Oded Galor for pointing this out.

level of national income determine the distribution of income? However, empirical support for this hypothesis has been mixed. For a review of this line of literature, I refer the reader to chapter 3 in Fields [2001].

Another line of literature asks the question in the reverse direction: how does income distribution affect the economy's ability to grow? Earlier theoretical analyses such as Loury [1981] generate an ergodic process converging to a unique income distribution regardless of initial distribution. Two theoretical papers bring a different light to this economic issue, Galor and Zeira [1993], and Banerjee and Newman [1993]. In contrast to earlier works, the models in these two papers exhibit multiple steady states, giving rise to a non-ergodic process (that means, initial condition matters for convergence). Although Banerjee and Newman also ask about initial distribution of income on economic performance, their interests shift more towards institutional change and occupational choice. Their conclusion, nonetheless, remains similar to Galor-Zeira: people with initially higher wealth will tend to choose more profitable occupation and be better off. So in a way, both of these models tell a story of how the economy will converge to heterogeneity not homogeneity in the long run, in the presence of credit constraints. A similar story is fleshed out in Aghion, Caroli, and Garcia-Penalosa [1999] using the "new growth theory" framework. Lastly, it is worth mentioning another literature studying the linkage from inequality to growth from a political economic perspective. Alesina and Rodrick [1994] elegantly demonstrates that high inequality leads to more redistributive policy and thus lower growth, because the policy discourages capital accumulation.

Danny Quah [1996] discusses some methodologies and works that have attempted to address this "twin-peak" limit in income distribution convergence.

Empirically, Quah [1997] and Jones [1997] document such tendency for the world income distribution to converge to a bi-modal limit. Evidence in Bourguignon and Morrison [2002] also bolsters the points about worsening inequality, and possibly a convergence to a bimodal distribution of income. In contrast, recent contribution by Sala-i-Martin [2006] shows that the world is converging, not diverging. He uses evidence from micro-level data as well as macro level data to reconcile different outcomes that may arise with different type of data sets. The debate continues, nonetheless. In the end, it may be impossible for the data and researchers to distinguish unimodal and bimodal world from one another.

The chapter is structured as follows. Section 2 reviews the original setup of Galor-Zeira model. I choose to present the version of the model as done in Basu [1997]. However, all interpretations and key elements remain identical to those done by the original authors. Section 3 delves directly into to the heart of the chapter, characterizing lump-sum taxes and transfers that will put every individuals on the prosperous path. In section 4, I add political consideration, analyzing simple political process and show that future still looks bleak. Section 5 discusses some lessons learned from my exercise and what policymakers should keep in mind. Finally, Section 6 concludes.

## 3.2 Galor-Zeira's Model

To avoid excess repetitions to Galor-Zeira's original analysis, I will not provide detailed descriptions of the economic environment here. Interested readers are referred to the original work for a complete motivation of variables.

The economy is small and open. Time is discrete and is indexed by  $t = 1, 2, \dots$

There is only one good in the economy, which gets produced by two technologies requiring either skilled or unskilled labor. One technology uses capital and skilled labor while the other uses only unskilled labor with a constant marginal product. These two labor markets generate two different wages,  $w_s$  and  $w_n$  for skilled and unskilled, respectively. Since this is a small-open economy, the interest rate  $r > 0$  on saving is taken as exogenous. The capital market is imperfect in the sense that individuals can borrow at  $i > r > 0$  while firms borrow at  $r$ .<sup>3</sup> This fixes the level of capital level in production as well as the two labor wages. Individuals inhabit for two periods in an overlapping-generation manner. I assume there are  $L$  dynasties, each consists of exactly 1 adult and 1 child. To give positive and measurable significance to each individual, I assume a finite number of  $L$  (in contrast to the original continuum of measure  $L$ ). Thus for any period  $t$ , the population size is  $2L$ , and there is no population growth.

All individuals are born identical except for the inheritance each receives. Adults care about bequest they leave behind for the child,  $b$ , and the consumption level,  $c$ . Consumption-bequest decisions are made by the adults only, so I can either assume that individuals consume only in the second period or  $c$  already incorporates the relevant child's consumption. I assume all adults have the following utility function,

$$u = c^\alpha b^{1-\alpha},$$

where  $\alpha \in (0, 1)$ . My use of Cobb-Douglas and Galor-Zeira's original use of the log utility function are to ensure that a constant fraction  $\alpha$  of wealth is consumed while the rest,  $1 - \alpha$ , is bequeathed. All individuals are endowed with a unit of

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<sup>3</sup>Galor and Zeira attribute this imperfection in capital market to the cost of keeping track on each borrowers who could default. The firms are less flexible than individuals, so they face less credit constraint. Nonetheless, the assumption that firms borrow at  $r$  is not necessary yet I provide it for completeness.

labor in each period and supply it inelastically (either for school or for work).

The young child receives the bequest in the first period of life and only decide whether or not to invest in human capital. If she decides to invest, a fixed indivisible fee  $h > 0$  is incurred and she foregoes her first period of earning as an unskilled labor in order to earn  $w_s$  only in the second period as a skilled adult. If she decides not to invest, she earns  $w_n$  as unskilled labor in both periods. Any inheritance not used can be saved, earning an interest at rate  $r$  and any lack of funds for human-capital investment can be borrowed with interest rate  $i$ .

For any level of lifetime resources  $y$ , the indirect utility of an adult is simply  $V = ey$  where  $e = \alpha^\alpha(1 - \alpha)^{1-\alpha}$ . She will consume  $c = \alpha y$  and will bequeath  $b = (1 - \alpha)y$ . If a young child with inheritance  $x$  decides not to invest in education, her indirect utility and bequest function looks as follows

$$\begin{aligned} V^n(x) &= e[(1 + r)(x + w_n) + w_n] \\ b^n(x) &= (1 - \alpha)[(1 + r)(x + w_n) + w_n]. \end{aligned}$$

For an individual who decides to invest in human capital when young, the functional form of her indirect utility depends on the level of bequest received. If the initial inheritance exceeds the cost of human capital,  $x \geq h$ , she can earn an interest  $r$  on any left over funds. However, if initial inheritance  $x$  is less than cost of education  $h$ , she must borrow extra funds at interest rate  $i > r$  to finance her education. By either mean of finance, she becomes unambiguously skilled in the next period and earns  $w_s$  for her labor. The indirect utility and bequest function

of skilled individuals is given piecewise by

$$\begin{aligned} V^s(x) &= \begin{cases} e[(1+r)(x-h) + w_s], & \text{if } x \geq h \\ e[(1+i)(x-h) + w_s], & \text{if } x < h \end{cases} \\ b^s(x) &= \begin{cases} (1-\alpha)[(1+r)(x-h) + w_s], & \text{if } x \geq h \\ (1-\alpha)[(1+i)(x-h) + w_s], & \text{if } x < h. \end{cases} \end{aligned}$$

Depending on parameter values, I could have a unique equilibrium where everyone works as unskilled. These may be of some other theoretical interests,<sup>4</sup> but it is of no interest in the context of my objective. To have some individuals acquire some skills, I must have the reward from education is large enough.<sup>5</sup> In algebraic terms, this means

$$w_s - w_n \geq (1+r)(w_n + h).$$

In words, investing in human capital must yield benefits that outweigh the opportunity cost.

Under the assumption above, I can write the wealth dynamics of the economy, using bequest functions defined above, as

$$x_{t+1} = b(x_t) = \begin{cases} (1-\alpha)[(x_t - h)(1+r) + w_s], & \text{if } h \leq x_t \\ (1-\alpha)[(x_t - h)(1+i) + w_s], & \text{if } f \leq x_t < h \\ (1-\alpha)[(1+r)(x_t + w_n) + w_n], & \text{if } x_t < f. \end{cases}$$

To get to the structure of the celebrated long-run bipolar convergence, I further assume that  $(1-\alpha)(1+r) < 1 < (1-\alpha)(1+i)$ . The dynamic evolution of

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<sup>4</sup>See footnote 12 in Galor and Zeira [1993].

<sup>5</sup>Incentives matter for effective education, as discussed in Section 5.

wealth distribution is illustrated in Figure 3.1.<sup>6</sup> Notice that,  $f$  defines the level of bequest that satisfies the equation:  $V^n(f) = V^s(f)$ . Therefore, any inheritance higher than  $f$  would make payoffs to education attractive to the young child in this period. But, even if the child receives bequest higher than  $f$ , this does not guarantee that the dynasty will end up as skilled. Unfortunately, although the bequest is higher than  $f$ , it is not high enough because still lower than  $g$ . This dynasty will eventually collapse down to the unskilled steady-state. The reason is that in the region  $x \in (f, g)$ , each bequest to the next generation shrinks in size. This continues until the incentive to invest in human capital disappears entirely. Therefore, this economy has a tendency to converge to a bipolar society with two distinct groups: rich and poor. How many end up on either end will be determined by the economy's initial distribution of wealth (where wealth in the subsequent periods is reflected in the bequest level).

### 3.3 The Redistribution Scheme

Before I state and prove my results, it is important to first understand what one-shot income redistribution can and cannot do. Consider an economy with per capita wealth *higher* than  $g$ . Under this case, egalitarian distribution will do the

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<sup>6</sup>I list here the parameters of Figure 3.1 for reference

$$\begin{aligned}\bar{x}_n &= \frac{(1-\alpha)(2+r)w_n}{1-(1-\alpha)(1+r)} \\ f &= \frac{(2+r)w_n + (1+i)h - w_s}{i-r} \\ g &= \frac{(1-\alpha)[h(1+i) - w_s]}{(1+i)(1-\alpha) - 1} \\ \bar{x}_s &= \frac{(1-\alpha)[w_s - (1+r)h]}{1-(1-\alpha)(1+r)}.\end{aligned}$$

For brevity, in other parts of the chapter, I will simply refer to the variable, but not its complete form.



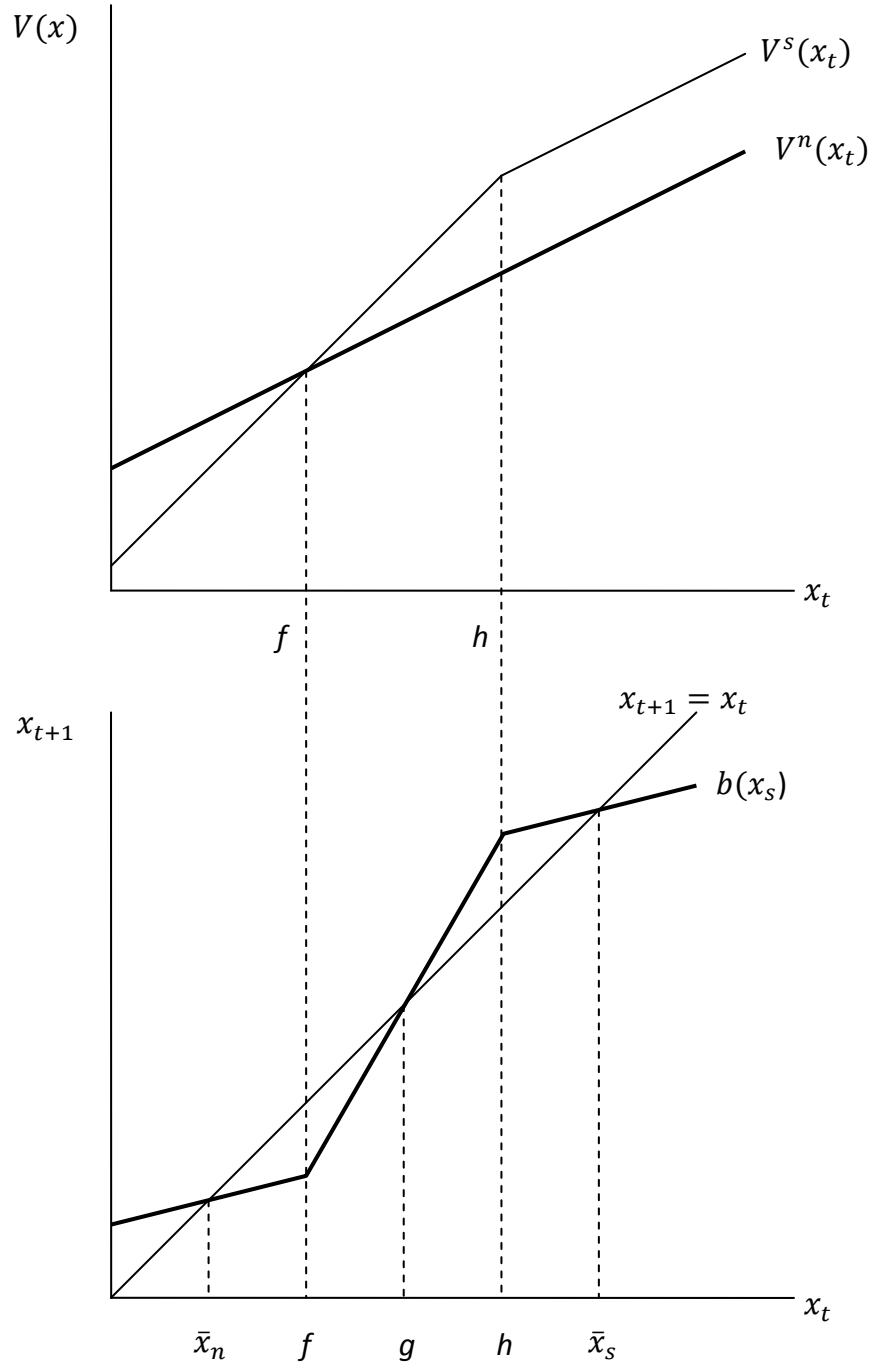


Figure 3.1: Top diagram shows indirect utilities of skilled ( $s$ ) and unskilled ( $n$ ) with respect to bequest level ( $x$ ). The diagram directly below depicts bequest dynamics. There are 3 steady-states:  $\bar{x}_n$ ,  $g$ , and  $\bar{x}_s$ . Their values are given in footnote 6 in the Chapter.

job of moving everyone to the higher steady-state. However, if we look at an opposite economy where per capita wealth is *lower* than  $g$  (which is very likely in the case of developing countries), egalitarian distribution will pull everyone down to the lower steady-state. Therefore, if we care about the economy's long run performance, measured in terms of wealth, then perfectly equal income distribution is not necessarily a good thing.<sup>7</sup> The best one-shot redistribution policy to fight poverty for any government in the economy is to minimize the headcount ratio, using  $g$  as the poverty line (see Basu [1997], p. 60-61).<sup>8</sup> This may even translate to taxing the poorer to give to the richer dynasties to get them over the threshold in an attempt to reduce the size of taxes and transfer. This leads to the conclusion that rich countries have a chance to fare better than poor countries from the very beginning. Both Basu [1997] and Galor and Zeira [1993] discuss policy implications with this model, recommending subsidizing education and lowering capital market friction (interest differential  $i - r$ ). None of the policy proposed, however, can totally eradicate poverty from this model. This is where my work fills in. I ask whether or not there is a way to entirely eradicate poverty *within* this model *without* external ad hoc features. My conclusion is yes, and the only tools required are lump-sum taxes and transfers.<sup>9</sup>

Assume there is a government (or a public institution) that could carry out costless lump-sum taxes and transfers to individuals. This same government could have carried out mass redistribution as described before. Eventually, what this

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<sup>7</sup>Remember that all individuals are identical except for the bequest they receive. I feel that the conclusion will be even stronger, if the people are heterogeneous in skills, and egalitarian redistribution destroys any economic incentive for any high-potential individuals. This is, of course, a subject for another project.

<sup>8</sup>If anyone with level of wealth below  $g$  is considered to be in poverty, then the head-count ratio is the fraction of population with wealth below that level.

<sup>9</sup>Admittedly, one may argue this is ad hoc in a way, and I partly agree. But I feel it is rather minimal since I do not introduce any new features that would alter the dynamics of the economy at all. All other policy recommendations are aimed at changing the structure of the economy.

government will want to do is to tax everyone and to *target* the transfer to a poor dynasty, getting it over the threshold. Then wait for economic growth to push all dynasties back to the tricker point where the government again carries out another taxes and target the transfer to another poor dynasty. The process (more like an algorithm) continues until everyone is above the threshold  $g$ . At this point everyone and all the generations thereafter have enough incentive to invest in human capital.

Let me now describe the redistribution scheme formally. For simplicity, let me first start with an economy where every dynasty is at the lower steady-state,  $\bar{x}_n$ . At this level, the economy has no investment in human capital at all. People simply work, save, and bequeath. The following proposition summarizes the result and states the sufficient conditions for the scheme to go through. I provide a proof in a longer but instructive fashion, outlining explicitly the steps for the government. The first sufficient condition requires that *aggregate* wealth in the economy is enough to support at least one dynasty over the threshold. I do not think of this as too strong of a requirement. The second condition imposes a restriction on the wage differential. Intuitively, it conveys that the skilled wage must be large enough to compensate for the missing unskilled wage as a consequence of the upgrading.

**Proposition 18.** *Suppose the economy has all  $L$  dynasties at the initial wealth of  $\bar{x}_n$ . If the economy's parameters is such that*

$$i.) \quad L\bar{x}_n > g$$

$$ii.) \quad \bar{x}_s - \bar{x}_n > g,$$

*then there is a sequence of lump-sum taxes and transfers on bequest to achieve full human-capital accumulation.*

*Proof.* I proceed in a sequence of steps. Define a positive small residual,  $\varepsilon > 0$ , from the two sufficient conditions *i.)* and *ii.)* such that  $\min \{L\bar{x}_n - g, \bar{x}_s - \bar{x}_n - g\} > \varepsilon > 0$ , then let  $\tilde{g} = g + \varepsilon$ . As it will turn out, the neighborhood  $\varepsilon$  will form a tricker point for the government's action. In addition, as will be clear later,  $\varepsilon$ -neighborhood will guarantee a finite implementation time.

*Step 1.* First, I will try to push one dynasty beyond the threshold  $g$ . Select a dynasty randomly.<sup>10</sup> Let  $T_1$  be the amount of transfer this dynasty receives and  $t_1$  be the tax paid by every dynasty in *Step 1*. The minimum necessary transfer is the amount just enough to get one dynasty pass  $g$  after paying taxes, or simply

$$T_1 = \tilde{g} - \bar{x}_n + t_1.$$

This transfer is financed by lump-sum taxes on all individuals (since all dynaties are identical at this point, there is no need for different taxes). The revenue from the tax collection must equal to the transfer above, which yields

$$Lt_1 = \tilde{g} - \bar{x}_n + t_1, \quad \text{or} \quad t_1^n = \frac{\tilde{g} - \bar{x}_n}{L - 1},$$

where  $t_1^n$  denotes tax on unskilled dynasties in the first step. Subtituting this back into the transfer equation, I find that

$$T_1 = \frac{L(\tilde{g} - \bar{x}_n)}{L - 1}.$$

Note that the burden of transfer is borne by  $L - 1$  dynasties. Condition *i.)* from the statement of the proposition guarantees that after-tax income of all dynasties will be non-negative.

*Step  $k + 1$ .* In this step, there are currently  $k$  dynasties above the threshold  $g$ , while  $L - k$  are still below it. So I want to allow two different levels of taxes:

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<sup>10</sup>Randomness is not necessary here. In fact, for this proposition, I could simply choose dynasty by its label,  $1, 2, \dots, L$ . Randomness will play a more important role in the political economy section below.

$t_{k+1}^n$  on the unskilled, and  $t_{k+1}^s$  on the skilled dynasties. I also need a little timing tool to help me characterize the algorithm completely. Let  $\tau$  be the *elapsed* time since the last distribution scheme (that is, since step  $k$  is completed). At the time of this redistribution, I will want to tax the skilled dynasties maximally, while still maintaining their position above the threshold  $g$ . This means

$$t_{k+1}^s = x_\tau^s - \tilde{g}, \quad (3.1)$$

where  $x_\tau^s$  denotes the bequest of the skilled dynasty at the elapsed time  $\tau$ . Again, let select a dynasty from those who are still below  $g$ . The lucky dynasty will receive a transfer equals to

$$T_{k+1} = \tilde{g} - x_\tau^n + t_{k+1}^n, \quad (3.2)$$

where  $T_{k+1}$  is the amount of transfer received by the unskilled,  $x_\tau^n$  is the bequest of the unskilled dynasty, and  $t_{k+1}^n$  is the amount of tax on all unskilled dynasties. Balanced budget of the government requires that tax revenue from both types must equal the single transfer to the selected unskilled dynasty, that is

$$(L - k)t_{k+1}^n + kt_{k+1}^s = T_{k+1}.$$

Using equation (3.1) and (3.3) in the balanced budget above and rearrange, I obtain the tax level on each unskilled dynasty as

$$t_{k+1}^n = \frac{(k + 1)\tilde{g} - x_\tau^n - kx_\tau^s}{L - k - 1}, \quad (3.3)$$

and the transfer to the lucky dynasty as

$$T_{k+1} = \frac{L\tilde{g} - (L - k)x_\tau^n - kx_\tau^s}{L - k - 1}. \quad (3.4)$$

Before redistributing, I must make sure that the after-tax income of the unskilled dynasty is non-negative, that is  $x_\tau^n - t_{k+1}^n \geq 0$ . To do that, I make use of equation (3.3) to get

$$(L - k)x_\tau^n + kx_\tau^s \geq (k + 1)\tilde{g}. \quad (3.5)$$

There is a good intuition behind this equation. It tells us that we can only redistribute if the aggregate income in the economy is enough to support the "after-redistribution" distribution that we want (having  $k + 1$  dynasties at  $\tilde{g}$ ). Lastly, I define for every  $k \in \{1, 2, \dots, L - 1\}$ ,

$$\tau_{k+1}^* = \min \{ \tau > 0 : (L - k)x_\tau^n + kx_\tau^s \geq (k + 1)\tilde{g} \}.$$

I think of  $\tau_{k+1}^*$  as the "wait time" before implementing the  $(k + 1)^{th}$  redistribution scheme.<sup>11</sup> The whole taxes and transfers scheme in Step  $k + 1$  can be defined as a function of this wait time. Once the condition in the definition of  $\tau_{k+1}^*$  is met, the time is prime for carrying out the scheme described prior by equation (3.1), (3.3), and (3.4).

*Step L.* Now all the  $L - 1$  dynasties are on the skilled path of the economy, while only one is below the threshold. This last step is similar to the ones above, with slight alterations. First, let me not tax the only poor household in the economy. Second, it may not be necessary to tax the richer household as far as all the way back to  $\tilde{g}$ . I only need to collect enough tax to push the last household pass  $\tilde{g}$ . These insights boil down to a supporting condition that

$$x_\tau^n + (L - 1)x_\tau^s \geq L\tilde{g},$$

or the aggregate income in the economy must be enough to support the entire country living beyond the threshold  $\tilde{g}$ . I define the wait time between *Step L-1* and *Step L* to be  $\tau_L^* = \min \{ \tau > 0 : x_\tau^n + (L - 1)x_\tau^s \geq L\tilde{g} \}$ . As noted above, tax on the poor is zero,  $t_L^n = 0$ . So the budget constraint gives a solution to the final amount of tax and transfer

$$t_L^s = \frac{\tilde{g} - x_{\tau_L^*}^n}{L - 1} \quad \text{and} \quad T_L = \tilde{g} - x_{\tau_L^*}^n.$$

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<sup>11</sup>I show below that this wait time,  $\tau_{k+1}^*$ , is finite for all  $k \in \{1, 2, \dots, L - 1\}$  in Lemma 19. Therefore, it justifies my use of the word "minimum" rather than "infimum."

The cost of my last redistribution scheme is spread over head of the skilled households. □

This leaves a positive note. At least, under the assumption in the model, club convergence is no longer a universal law like gravity. There is a room for improvement. I can draw some interesting lessons from equation (3.3). The higher the income level of each group, the less tax burden there is on the poor. This is because I need less transfer to fill in the gap between the poor's wealth and the threshold level. Since the rich group is a contributor, the higher their income, the easier the scheme is on the poor. So economic growth is an important ingredient in the effectiveness of this redistribution scheme. In addition, at the beginning of *Step  $k+1$* , I mention that I want to tax skilled household maximally. This is not a necessary requirement. Nonetheless, any dollar less in tax on the skilled dynasties implies a dollar increase in the tax on the unskilled dynasties or a longer wait time.

The following lemma shows that the entire implementation time of the redistribution above is finite. I prove this by showing that each wait time defined in Proposition 18 is finite, so the finite sum of all  $L$  wait times must indeed be finite. To prove this analytically, I utilize the two conditions in Proposition 18 and the convergence properties of the sequence of bequests.

**Lemma 19.** *Define  $\tau_1^* = 0$  as the wait time for the first step. Then for every  $k \in \{1, 2, \dots, L\}$ , the wait time  $\tau_k^* < \infty$ . Therefore, the total implementation time of the redistribution scheme constructed in Proposition 18 is finite, or, algebraically  $\sum_{k=1}^L \tau_k^* < \infty$ .*

*Proof.* First, I define  $\bar{x}_n \equiv \lim_{\tau \rightarrow \infty} x_\tau^n$  and  $\bar{x}_s \equiv \lim_{\tau \rightarrow \infty} x_\tau^s$  as the lower and the higher steady state in the model, respectively. Under the parametric assumptions, all

sequences of bequests converge to a limit point. Second, these sequences are converging from below, thus they are monotonically increasing. Recall that after a redistribution, the bequest of every dynasty is positioned below its limit point. Now, I define the aggregate income of the economy in *Step*  $k+1$  as another sequence by  $x_\tau^k = (L - k)x_\tau^n + kx_\tau^s$ . Then

$$\begin{aligned}\lim_{\tau \rightarrow \infty} x_\tau^k &= (L - k)x_\infty^n + kx_\infty^s \\ &= (L - k)\bar{x}_n + k\bar{x}_s \\ &> (k + 1)\tilde{g},\end{aligned}$$

where the inequality follows from using condition *i.)* and *ii.)* in Proposition 18 and also the definition of  $\tilde{g}$ . Since the sequence of aggregate income also converges monotonically to its limit,  $(L - k)\bar{x}_n + k\bar{x}_s$ , by definition of convergence, there exists a  $\tau_{k+1}^* < \infty$  such that for every  $\tau > \tau_{k+1}^*$ , I must have  $(k + 1)\tilde{g} < x_\tau^k < (L - k)\bar{x}_n + k\bar{x}_s$ . In words, any convergent sequence has a requirement that, for a time big enough but finite, the sequence must lie within a neighborhood around its limit point forever from then on. This proof remains valid for every step  $k \in \{0, 1, 2, \dots, L - 1\}$ . Finally, the finite sum of a collection of finite numbers is finite. So the total implementation time is finite.  $\square$

This result is rather heartwarming: we can, in fact, eradicate poverty (within this model) in finite time. Finite is still nonetheless too vague. Even though it is already a restriction from infinity, finiteness could still mean many things. Unfortunately, I do not have a result that will strengthen my notion of "finiteness" in this chapter. Despite these weaknesses, I will discuss what we can learn from this Lemma more thoroughly below.

So far, I have provided results based on only one initial distribution of wealth: everyone starts at the lower steady-state,  $\bar{x}_n$ . To address this issue, the next



proposition generalizes the idea developed in Proposition 18 to encompass arbitrary initial distribution of wealth. In proving the proposition, I use the following logic. If you could redistribute with any steps described in Proposition 18, do so. If you cannot, the bequest dynamics will take you to the point where you eventually could. This implies additional wait time, but only in the first step, and still is finite.

**Proposition 20.** *Any economy with the parameters satisfying the two conditions*

$$i.) \quad L\bar{x}_n > g$$

$$ii.) \quad \bar{x}_s - \bar{x}_n > g,$$

*can escape the poverty trap with a sequence of lump-sum taxes and transfers, regardless of the economy's initial distribution of wealth.*

*Proof.* Notice that, if the economy has a positive number of dynasty, say  $k$ , beyond  $\tilde{g} = g + \varepsilon$ , defined properly above in Proposition 18, then the economy can immediately proceed with *Step*  $k + 1$  of Proposition 18. There is no further work to be done for this case. The new work that requires a verification is when the initial distribution places all individuals below  $\tilde{g}$ , but not at the same wealth level. Let  $x_{t,l}$  be the bequest of dynasty  $l \in \{1, 2, \dots, L\}$ . All of  $x_{t,l}$  are smaller than  $\tilde{g}$ . Select a dynasty  $l^* \in \{1, 2, \dots, L\}$  to receive the transfer  $T_{1,l^*} = \tilde{g} - (x_{\hat{\tau}_1^*, l^*} - t_{1,l^*})$ , where  $\hat{\tau}_1^*$  denotes the first elapsed time. The tax revenue collected must be enough to finance the transfer,  $\sum_{l=1}^L t_{1,l} = \tilde{g} - x_{\hat{\tau}_1^*, l^*} + t_{1,l^*}$ , which gives the equation

$$\sum_{l \neq l^*} t_{1,l} = \tilde{g} - x_{\hat{\tau}_1^*, l^*}.$$

I cannot redistribute unless, for every  $l \in \{1, 2, \dots, L\}$ ,  $x_{\hat{\tau}_1^*, l} - t_{1,l} \geq 0$ . Summing across  $l \neq l^*$ , I get  $\sum_{l \neq l^*} x_{\hat{\tau}_1^*, l} \geq \tilde{g} - x_{\hat{\tau}_1^*, l^*}$ . In light of Proposition 18, it is clear

that I will want to wait until  $\sum_{l=1}^L x_{\widehat{\tau}_1^*, l} \geq \widetilde{g}$ , before proceeding with the first redistribution. Condition *i.*) guarantees that such  $\widehat{\tau}_1^*$  exists and is finite. From then on, I simply follow steps outlined in Proposition 18.  $\square$

Will this proposition alter the result of finiteness above? No. The finiteness shown in Lemma 19 applies to all initial wealth distribution satisfying the sufficient conditions *i.) – ii.)*. Although, one type of initial distribution may just take longer than the other.

In this section, I provide a new possibility to overcome the poverty trap. My results show that the poor economies could do better, with the help of redistribution and economic growth. However, this requires a (sequence of) benevolent government(s) with enormous foresight to implement the proposed redistribution scheme. In the next section, this long-term political commitment is investigated.

### 3.4 Political Obstacles to Development

In this section, I attempt to shed light on whether or not the redistribution policy will survive the political process. Consider a political institution, possibly democratic, with a parameter  $\delta$  denoting the political power of the skilled dynasties. For example, when  $\delta = 30\%$ , the skilled dynasties only need to form at least 30% of the voting population in order to have the decisive power to implement their preferred policy. A balanced (two-group) democratic institution would have  $\delta = 50\%$ .<sup>12</sup>

The intuition for the next proposition is the following. Because the transfers only benefit the poor dynasties, there is no incentive for the rich to continue with

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<sup>12</sup>In a two-group democracy, for example,  $\delta$  could be 50% for just majority or 60% for supermajority.

the redistribution scheme. I view this as related to the idea of Kuznets's hypothesis (see Kuznets [1955]): process of transition from developing to developed must go through a period of high inequality in the economy. This transition also brings about a shift in political power. Once the rich dynasties garner enough political support, the redistribution program is forever on a halt. I formally organize this line of arguments in the following proposition.

**Proposition 21.** *For any political institution with  $\delta < 100\%$ , the redistribution scheme will not survive in the political process.*

*Proof.* Because the model reduces heterogeneity down to two groups, those above and below  $g$ , I focus on the two sets of indirect preferences. When thinking about indirect utilities, I note that they are risk-neutral with respect to uncertainty in wealth (recall that  $V = ey$ , where  $y$  is lifetime resources). Therefore, it suffices for me to simply focus on the expected lifetime resources of the pivotal voter. But these depend linearly on the level of bequest, so instead I can just focus on what happens to individual bequest.

Clearly if the pivotal voter has bequest above  $g$ , there will be no redistribution. The skilled dynasties only pay taxes and receive no benefits. That means, as soon as those above the threshold  $g$  have gathered enough political leverage, the redistribution scheme stops. The termination is imminent because the scheme keeps adding to the size of the skilled group. The question now is, will the redistribution scheme ever takes place at all? I analyze this in details.

Assume all dynasties are below  $g$ , as in *Step 1*, and a dynasty is chosen with equal probability to receive the transfer. So any child's expected bequest is  $x_t^n - t_1 + \frac{1}{L}T_1$ , where  $t_1$  and  $T_1$  are uniform tax and targeted transfer, respectively. For simplicity, let me consider the homogeneous initial distribution with everyone at

the lower steady-state. Let  $\tau$  denote the time for taxing, then use  $t_1 = \frac{\tilde{g}-x_\tau}{L-1}$  and  $T_1 = \frac{L(\tilde{g}-x_\tau)}{L-1}$  from Proposition 18, I see that expected bequest remains exactly at  $x_\tau$ . Whether or not there will be redistribution depends on how indifferent voters vote. If I assume that they vote the brighter future, then the redistribution scheme will begin.<sup>13</sup>

For *Step*  $k + 1$ , let  $\tau_{k+1}^*$  denote the time of taxing. The expected wealth of an unskilled dynasty is  $x_{\tau_{k+1}^*}^n - t_{k+1} + \frac{1}{L-k}T_{k+1}$ . Using equation (3.1), (3.4), and (3.3), I find that the expected wealth is higher with redistribution than without, that is

$$x_{\tau_{k+1}^*}^n + \frac{k}{L-k}(x_{\tau_{k+1}^*}^s - \tilde{g}) > x_{\tau_{k+1}^*}^n.$$

The more numerous the skilled dynasties are (as denoted by  $k$ ), the higher the benefits the unskilled can expect from the transfer. Hence, as long as the political power lays in the hands of the unskilled dynasties, the redistribution scheme will continue. Of course, only so until the political power changes hand.  $\square$

The mechanism that stops economic development in my case is economic losers not political losers as in Acemoglu and Robinson [2000]. However, the process of development comes from devising a dynamic plan of income redistribution instead of technology. Political losers in this case are the unskilled dynasties, who lose power as soon as the pivotal voter moves over the threshold.

Unfortunately, the proposition also implies that all forms of governance will not take the economy to the bright future with this policy. This includes dictatorship and democracy. One can think of the dictatorship regime as having  $\delta = \frac{1}{L}$ . The dictator will have an incentive to only push his dynasty over the threshold, and may

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<sup>13</sup>It can be shown that, for initial distribution with heterogeneous poor households, those with the tax rate  $t_{1,l^*} \leq \frac{\tilde{g}-x_{\tau_{1,l^*}^*}}{L-1}$  will vote for the policy. Thus the redistribution is more likely to be adopted in very poor economies.

be some other selected few. Even if the dictatorship regime is overthrown (either by domestic or international forces) and the economy transforms into a democracy, the bitterness of Proposition 21 will eventually kick in. I should point out that, in spite of these results, I believe that democracy at least will reduce more poverty than dictatorship.

Admittedly, this section reaches a negative conclusion for both democracy and dictatorship. A simple hypothesis here is that the rate at which poverty is reduced by policies will diminish as the economy transits from its developing to developed status due to a shift in political sentiment. The policymakers of any country must be careful with political conflicts arising from inequality along the path of development. Hence, the medium-income economies could remain "medium" for a long time if the domestic politics focus heavily on redistribution. The key question that I have now is, how will these medium-income economies reconcile its domestic conflicts and pull itself out of this transitional trap? Many east and southeast Asian countries have successfully done so. But many other countries, including those from the same region, still struggle to cross over to the threshold. I offer some further discussions in the next section.

### **3.5 Discussion**

I assume that there is a public institution that could costlessly carry out lump-sum taxes and transfers. Realistically, there is nothing near such a frictionless feature. Tax and transfer policies are often costly, both economically and politically. However, even if such tools were to approximately exist, they fall short of being perfect. A dollar of transfer would often fail to generate a dollar worth

activity in the developing world (see, for example, Easterly [2001]).

On the method of redistribution, I have one point to re-emphasize. Although, in the chapter, I always tax the skilled dynasties maximally, I am always careful not to destroy their incentive to invest in human capital (and the incentive of their later generations). I do so by maintaining their position above the threshold level  $\tilde{g}$ . So we must not destroy any incentive of the most efficient worker in the economy, since effective redistribution scheme needs them. It is worth mentioning that I allow no labor emigration. So there is no braindrian in the economy. If, however, I allow individuals to choose either to be in or out of the economy, enforcing taxes on the skilled dynasty will be harder to achieve. And frankly, without them, the whole redistribution scheme is an impossible task.

Finiteness of my redistribution scheme is a plus, but still is not the reason to celebrate. The set of possible implementation time turns out to equal to the set of positive reals. Further works need to be done here. There are a few additional things I could say to improve relatively the size of this finite time. Recall that  $g = \frac{(1-\alpha)[h(1+i)-w_s]}{(1+i)(1-\alpha)-1}$ , while  $\tilde{g} = g + \varepsilon$ . Any policy measure that reduces  $g$  will improve the time it takes to complete the whole steps. Simple differentiation reveals that  $\frac{\partial g}{\partial i} = \frac{(1-\alpha)[w_s-h]}{[(1+i)(1-\alpha)-1]^2} > 0$ . So a reduction in the cost of investment in human capital,  $h$ , a decrease the borrowing interest rate, and an increase in wage of skilled workers,  $w_s$ , all help lowering  $g$  thus lowering the implementation time. They reduce the amount of time necessary for the economy to accumulate wealth enough to support the after-tax redistribution. Relevant policies will include subsidizing education, providing cheap and accessible education funds for the poorer class, and encouraging R & D investment.<sup>14</sup> Note that skill-biased technological progress may

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<sup>14</sup>I am implicitly under the assumption that improvement in technology lifts the income the skilled faster relative to those unskilled. See, for example, Krusell et. al. [2000]. In fact, I only require an increase in  $w_s$ , so relativity does not matter here. See Krusell et. al. [2000].

not be detrimental if the right redistribution mechanism is at play. This generates a premium as an additional incentive to go to school and be educated. As argued convincingly in chapter 4 of Easterly [2001], incentive for education matters. "Creating skills where there exists no technology to use them is not going to foster economic growth [page 73]."

I have mentioned that, in contrast to Acemoglu and Robinson [2000], my political economy obstacle to development is driven by economic, not political losers. However, if I push along line of this Acemoglu et. al. [2001]'s idea of political losers, the group of unskilled dynasties may, in fact, decide not push the decisive voter pass the threshold. In that way, taxation of the rich continues forever (and may eventually turns into expropriation), while the revenues collected will be divided among the unskilled dynasties. Such action is conceivable, and will eventually destroy all the incentives to invest in the future and encourage a flight of brains and capital to elsewhere. Nonetheless, because I do not endogenize the political selection mechanism here, this sequence of thoughts needs further and rigorous exploration. Without that, this is simply a matter of an educated speculation.

Lastly, there is a strong stylized fact that skilled laborers tend to provide positive externality to one another. For example, a celebrated work of Kremer [1993] argues that skilled labor provide positive externality to one another. Kremer and Maskin [1996] also finds further empirical evidence supporting this position. That means, the more skilled labors a country has, the more productive the skilled labors are. If labors are paid according to their productivity, this translates a positive relationship between the size of skilled labor force and their wage, in contrast to the usual diminishing returns. This feature is absent from the Galor-Zeira's analysis, and subsequently also from mine. If there is an increasing returns to being a skilled

labor, then the political obstacle from the skilled dynasties would be weakened.

I still caution those policymakers fond of redistribution. Unfortunately, if we believe even slightly in the logic of the political economy section (Proposition 21), we must conclude that all forms of government cannot take the economy to the bright future through redistribution alone. Moreover, a heavy reliance on the redistribution scheme would generate a brain flight and rid the economy of any chance to escape poverty. Prosperity of the economy should, at the end, come from institutional reforms and technological advancement, not from redistribution alone.

### **3.6 Conclusion**

Previous studies on poverty traps show that egalitarian redistribution may have an adverse effect on the economy by pulling everyone down below the threshold for the trap. The chapter sets out to address simple, but subtle, questions: Can redistribution help all the poor escape the poverty trap? And if so, will the redistribution policy survive to fulfill its purpose from the political process? To shed lights on the answers, I use on a particular model of poverty traps studied in Galor and Zeira [1993]. Despite the emphasis on a particular model, my result on redistribution can easily be generalized to other models. In return for the specificity, however, I get to draw deeper insights into the issue at hands with well-specified behavioral and economic parameters.

It turns out that, indeed, a careful dynamic redistribution can be an escape for the economy out of a poverty trap. In the chapter, I carefully crafted a scheme of taxes and transfers that get this result. First, I push a dynasty pass the poverty



threshold, then I patiently wait for wealth accumulation from both the poor and the rich group. Once sufficient amount of wealth is available in the economy, I again push another dynasty over the threshold. I repeat this process until every dynasty is above the threshold.

Furthermore, I show that my redistribution scheme can achieve the result in finite amount of time. As aforementioned, finite is still too vague. Although one can never precisely pin down the actual day and time that the redistribution scheme will succeed, I believe that an efficiency result should nonetheless be within reach of future work. Can we do better than the algorithm here, for example, pushing more than just one dynasty at a time above the threshold? How much control do we have over the implementation time by varying different parameters in the model?

As for my political economic inquiry, I have not endogenized the political choice of the redistribution policy. I take the redistribution scheme above as given and analyze whether or not it would be adopted. Unfortunately, the redistribution will not survive the political struggle. When the economy transits from a poor to a richer status, the political influence of the wealthier dynasties will also grow. Eventually, the redistribution scheme will be stopped. Future works may be interested in modeling the political process in the similar fashion to those of Acemoglu and Robinson [2006]. Such a framework will allow for an endogeneous determination of tax rates, which I have no reason to believe that it will be equal to my analysis here. It will further allow for choices of different regimes and actions, for example, revolution, expropriation, and taxes.

Using this essay, I argue for a policy that balances both growth and redistribution objective. I also support an active role of the government in providing

education subsidies and education funds, while making sure that the rewards to being skilled remains high. Effective redistribution, as a tool against poverty, requires inputs from the rich (and skilled) households. Prosperity of the economy should, at the end, come from institutional supports and reforms in congruence with technological advancement, not from redistribution alone. In light of recent trend in globalization and skill-biased technological progress, policymakers face tougher challenges in taming inequality while continue promoting growth. Trying to strike a balance between the two objectives will remain an art, not science, for the policymakers, and a lifetime research agenda for the academia.

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APPENDIX A  
APPENDIX OF CHAPTER 1

## A.1 Proofs of the Propositions

*Proof.* (**Proposition 2**) I prove this proposition in two steps. First, I show that if the young in this period will try to influence the identity of the decisive voter in the next period, they prefer to put themselves as the majority instead of other group. This may seem obvious, but it requires a rigorous verification. By using the inequalities from Appendix A.2 and A.3, there are two cases to consider. The proof uses the same logic in all cases: the binding restriction for having the unskilled as the largest in the next period turns out to be a subset of the restrictions for having the old as the largest group in the next period. However, to facilitate the proposition using the indirect utilities, I know that  $\frac{\partial V^s}{\partial \sigma_t} > 0$  and  $\frac{\partial V^u}{\partial \sigma_t} > 0$  leading to  $\sigma_t = 1$ , as long as  $\mu_t > 0$ .

- (i) When  $s_t \leq \frac{1}{1+n}$ , to have the old dominate the largest in the next period, I have  $n - (1+n)s_t \leq \mu_t \leq \frac{1-(1+n)s_t}{m}$ . With  $\sigma_t = 1$ ,  $\frac{\partial V^h}{\partial \mu_t} > 0$  for both  $h \in \{s, u\}$ , so  $\mu_t^h = \frac{1-(1+n)s_t}{m}$ . This is higher than the level necessary to have the unskilled dominate,  $n - (1+n)s_t$ .
- (ii) When  $s_t > \frac{1}{1+n}$ , for next period old to dominate the unskilled requires  $n - (1+n)s_t \leq \mu_t$ . But the condition for this case implies  $n - (1+n)s_t < 0$ , so the inequality is always true. I focus solely on the old dominating the skilled in the next period, which requires  $\mu_t(1 - (1+m)\sigma_t) \geq (1+n)s_t - 1$ . Notice that there is a bound to which  $\sigma_t$  could go up to. Having the unskilled as the largest in the next period requires that  $\mu_t(m - (1+m)\sigma_t) \geq (1+n)s_t -$

$n$ . It turns out that the latter condition is simply a strict subset of the first inequality, which means that they can always get strictly higher skilled composition with the same immigration quota. Therefore, it is better for the skilled young to place itself as the decisive old in the next period .

Lastly, to tie down the proposition, I must show that the policy choices provided in the proposition are optimal for each group. In addition, I must prove that there is an incentive for the young to restrict their choice of immigration today, in order to place themselves as the decisive old in the next period. I consider first the preference of the old,

$$V_t^o = T_t^o = \frac{\tau_t(1 - \tau_t)^\varepsilon A_t^{1+\varepsilon} \left\{ (s_t + \sigma_t \mu_t) (w_t^s)^{1+\varepsilon} + (1 - s_t + \sigma_t(1 - \mu_t)) (w_t^u)^{1+\varepsilon} \right\}}{1 + \mu_t + \frac{1 + \mu_{t-1}}{1 + n + \mu_{t-1}(1 + m)}}.$$

Differentiation reveals that  $\frac{\partial V_t^o}{\partial \sigma_t} > 0$ , when  $\mu_t > 0$ , and  $\frac{\partial V_t^o}{\partial \mu_t} > 0$ , when  $\sigma_t$  is sufficiently high. This gives  $\mu_t^o = \sigma_t^o = 1$ . Differentiating with respect to tax,  $\tau_t$ , and setting equal to zero yield  $\tau_t^o = \frac{1}{1+\varepsilon}$ , the Laffer point.

Next I turn my attention to the young. Since wages are kept fixed and the skilled will never implement a positive tax on themselves, there is no channel for the skilled to be affected by immigration in this period. However, if the skilled weighs in the future, what they could gain appears more vivid. Consider the skilled young's preference, under the assumption that  $w^s > w^u$ , the skilled voters want no tax, hence  $\tau_t^s = 0$ , which reduces their preference down significantly to

$$V_t^s = \frac{(A_t w_t^s)^{1+\varepsilon}}{1 + \varepsilon} + \beta T_{t+1}.$$

Because immigration in this period does not affect anything else in period  $t$ , the skilled young consider the benefits they could reap in the next period from skilled

immigrants. These are exactly the preference of the old generation in period  $t + 1$ . Nonetheless, if they maximally open the economy to skilled immigration,  $\sigma_t = 1 = \mu_t$ , the skilled voters will dominate in  $t + 1$ , and  $T_{t+1} = 0$ . Therefore, the skilled must restrict some skilled immigrants to ensure the existence of next period's transfer. Consider first when  $s_t \leq \frac{1}{1+n}$ , I know from above analysis that  $\mu_t^s = \frac{1-(1+n)s_t}{m}$  and  $\sigma_t^s = 1$ . When  $s_t > \frac{1}{1+n}$ , I have a complication; the skilled group in this period is growing too fast. In order to *not* place the decisive power in the hands of the next period's skilled, it becomes necessary to restrict both skill composition of the immigrants and immigration volume. The eventual choice can be specified implicitly as the solution to the exercise

$$\begin{aligned} \langle \sigma_t^s, \mu_t^s \rangle &= \arg \max_{\sigma_t, \mu_t} V_t^s = \frac{(A_t w_t^s)^{1+\varepsilon}}{1+\varepsilon} + \beta T_{t+1}^o \\ \text{s. t.} \quad &(1+n)s_t - 1 \leq \mu_t(1 - (1+m)\sigma_t). \end{aligned}$$

The restrictive condition in the optimization problem comes from trying to have the old dominate the skilled young in the next period. When the solution to the above problem is interior, I can describe it by

$$-\frac{\frac{\partial V^s}{\partial \sigma_t}}{\frac{\partial V^s}{\partial \mu_t}} = \frac{\mu_t^s(1+m)}{1 - (1+m)\sigma_t^s}.$$

Possible corner solutions are  $\langle \sigma_t^s, \mu_t^s \rangle = \langle 0, (1+n)s_t - 1 \rangle$  and  $\langle \sigma_t^s, \mu_t^s \rangle = \left\langle \frac{2-(1+n)s_t}{1+m}, 1 \right\rangle$ .

Turning now to the preference of the unskilled young, differentiating the unskilled's indirect utility with respect to  $\tau_t$ , holding other policy constant yields

$$\left(1 - \varepsilon \frac{\tau_t^u}{1 - \tau_t^u}\right) \left( \frac{(s_t + \sigma_t \mu_t) \left(\frac{w_t^s}{w_t^u}\right)^{1+\varepsilon} + 1 - s_t + (1 - \sigma_t) \mu_t}{1 + \mu_t + \frac{1 + \mu_t - 1}{1 + n + \mu_t - 1(1+m)}} \right) = 1,$$

assuming interior solution. This can be solved for  $\tau_t^u$  in the form described in the proposition with

$$J(s_t, \mu_{t-1}, \mu_t, \sigma_t) = \frac{(s_t + \sigma_t \mu_t) \left( \frac{w_t^s}{w_t^u} \right)^{1+\varepsilon} + 1 - s_t + (1 - \sigma_t) \mu_t}{1 + \mu_t + \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}}.$$

Notice that the case  $s_t > \frac{1}{1+n}$  cannot happen if the unskilled group is the largest. As for the policy choice on immigration, I must compare utilities that the unskilled voters would receive if they implement the ideal policy irrespective of any political consideration about the next period versus if they implement more restrictive policies in order to put themselves as the decisive old in the next period. I denote with  $\widehat{T}_t$  the transfer this period with  $\mu_t = 1 = \sigma_t$ , and I denote with  $T_t^u$  the transfer they would have gotten with  $\sigma_t = 1$  and  $\mu_t = \frac{1-(1+n)s_t}{m}$ . Such comparison concludes that the unskilled young will with old some immigration quotas if and only if

$$0 < T_t^u + \beta T_{t+1}^o - \widehat{T}_t = \Psi \left( \sigma_t = 1, \mu_t = \frac{1 - (1+n)s_t}{m} \right).$$

Can this condition be satisfied? I believe so. I know that  $T_t^o > \widehat{T}_t > T_t^u$  (where  $T_t^o$  is the transfer level preferred by the old in period  $t$ ), if the level of transfer preferred by the old in the next period is close to the transfer preferred by the old voters in this period ( $T_{t+1}^o \approx T_t^o$ ), then there is  $\beta$  such that this inequality will undoubtedly hold.  $\square$

*Proof. (Proposition 4)* First, I recognize that, in any period  $t$ , there are three possible states of the world: either the skilled workers, unskilled workers, or old retirees constitute the largest group in the economy. When  $s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$ , the skilled group is the largest bloc and unskilled and the old voters will collude their votes to the unskilled representative. When the skilled voters are not the absolute majority, whether they vote  $e_t^{s*} = "s"$  or  $"u"$ , the policies preferred by the unskilled will be implemented. None has any incentive to change their voting



strategy, given the others' voting strategy. The case where the old retirees are the largest can be argued in a similar manner. When the unskilled group is the largest and votes for its representative candidate, the two groups' votes have no pivotal power (under assumption 5, no weakly dominated strategy).  $\square$

*Proof. (Proposition 5)* Formally, the tax level,  $\tilde{\tau}$ , is defined implicitly by the equation

$$\frac{(A_t w^u)^{1+\varepsilon}}{1+\varepsilon} = \frac{(A_t w^u (1-\tilde{\tau}))^{1+\varepsilon}}{1+\varepsilon} + \frac{\tilde{\tau}(1-\tilde{\tau})^\varepsilon A_t^{1+\varepsilon} ((s_t + \sigma_t \mu_t) (w^s)^{1+\varepsilon} + (1-s_t + (1-\sigma_t) \mu_t) (w^u)^{1+\varepsilon})}{1 + \mu_t + \frac{1+\mu_t-1}{1+n+\mu_{t-1}(1+m)}}.$$

I know that such a tax policy exists, because the payoff in this period (holding the transfer in the next period constant) to the unskilled is maximized at the preferred policy and zero at  $\tau = 1$ . Therefore, at some  $\tilde{\tau} \in [0, 1]$ , the equality will hold. Any tax higher than this cut-off level would appear as too "redistributive" (or "extractive") to the unskilled voters and, on the contrary, any tax below this, they still benefit from the welfare state. When  $\frac{1}{1+\varepsilon} \leq \tilde{\tau}$ , the unskilled workers have an incentive to side with the old retirees. If  $s_t < \frac{1+\frac{n-m}{2}}{1+n}$ , then opening of the economy fully to the skilled immigrants will not place the skilled young as the *absolute* majority in the next period. If the old voters always vote for themselves, the unskilled young can either vote for themselves, when they are the largest, or vote for the old and enjoy some redistribution. When the victory goes to the old candidate, under the parameters I described, the next period's largest group will be skilled. However, the collusion of votes between the unskilled and the old voters will still prevail in the next period and the redistribution continues at the rate preferred by next period's old (which will be ideal for this period's young). When  $s_t \geq \frac{1+\frac{n-m}{2}}{1+n}$ , voting for the old will place the next period's skilled voters as the absolute majority, guaranteeing no redistribution in the next period. In this

case, the unskilled voters will vote for their candidate and, with the help of the skilled voters, get to implement their preferred policies.  $\square$

*Proof. (Proposition 6)* The cut-off tax level  $\tilde{\tau}$  is defined in the proof for Proposition 5. With  $\frac{1}{1+\varepsilon} > \tilde{\tau}$ , both the skilled and unskilled young try to avoid the policies of the old. Both will vote for the skilled candidate whenever the old is the largest group, thus getting the preferred policies of the skilled implemented. On the contrary, the old voters will try to avoid the policies of the skilled young, by voting for the unskilled candidate when the skilled voters are the largest group, hence resulting in the policies preferred by the unskilled workers.  $\square$

*Proof. (Proposition 7)* Consider first the preference of the unskilled young. The unskilled wage is given by equation (1.11), which responds positively to  $\sigma_t$ . So there are three channels to which the composition of immigrants this period affect their preference: wage, this period's transfer, and next period's transfer. All three channels benefit from increasing this period's skilled immigrants. Differentiating the unskilled's indirect utility with respect to  $\tau_t$ , holding other policies constant yields

$$\left(1 - \varepsilon \frac{\tau_t^u}{1 - \tau_t^u}\right) \left( \frac{1 - s_t + (1 - \sigma_t)\mu_t}{(1 - \alpha) \left(1 + \mu_t + \frac{1 + \mu_{t-1}}{1 + n + \mu_{t-1}(1 + m)}\right)} \right) = 1,$$

assuming interior solution. This can be solved for  $\tau_t^u$  in the form described in the proposition with

$$K(\mu_t, \sigma_t, s_t, \mu_{t-1}) = \frac{1 - s_t + (1 - \sigma_t)\mu_t}{(1 - \alpha) \left(1 + \mu_t + \frac{1 + \mu_{t-1}}{1 + n + \mu_{t-1}(1 + m)}\right)}.$$

For period  $t$ , the unskilled voters seem to prefer  $\sigma_t^u = 1$ . However, before drawing a definite conclusion, I must compare the two levels of utility generated by completely allowing for maximum possible skilled immigrants and restricting them in order

to be the decisive voter in the next period. The unskilled young would try to influence next period's identity of the decisive voter if

$$V_t^u(\sigma_t = 1, \mu_t = \frac{1 - (1+n)s_t}{m}, \tau_t^u) > V_t^u(\sigma_t = 1, \mu_t = 1, \tau_t^u).$$

So I could define an auxiliary function as their difference  $\hat{\Psi}(\sigma_t = 1, \mu_t = \frac{1-(1+n)s_t}{m}) = V_t^u(\sigma_t = 1, \mu_t = \frac{1-(1+n)s_t}{m}, \tau_t^u) - V_t^u(\sigma_t = 1, \mu_t = 1, \tau_t^u)$ .

As for the native-born skilled workers, the situation becomes increasingly more complex. Their preferred tax rate for this period is still zero,  $\tau_t^s = 0$ . However, skilled immigrants generate both cost and benefit to the native-born skilled. They provide a direct labor market competition, forcing the current skilled wage down. Ideally, the skilled natives would have preferred all unskilled immigrants for this reason. However, skilled immigrants also provide future benefits through higher transfer in the next period. This conflict makes their policy choices unclear. One thing is clear, nevertheless, the skilled will always believe that there is positive future benefit once they retired. To see this, consider if they think the skilled will for the majority next period, hence  $T_{t+1}^s = 0$ . Then the only gain from immigrants would come from bringing as many unskilled in as possible to lift up the wage. That leads to  $\sigma_t = 0$ , and  $\mu_t = 1$ , which in turn will make the unskilled voters the largest in the next period. But this produces a contradiction to the initial belief of the skilled. Hence I know that, if the skilled voters are the largest in this period, next period's decisive voter will either be the unskilled young or the old. With this information, there are two problems to solve. First, if next period's decisive voter is unskilled, then the utility accrued to the skilled is

$$V_t^{s|u} = \max_{\sigma_t, \mu_t \in [0,1] \times [0,1]} V_t^s = \alpha^{1+\varepsilon\alpha} (1-\alpha)^{\varepsilon(1-\alpha)} A_t^{1+\varepsilon} \left( \frac{1-s_t + \mu_t(1-\sigma_t)}{s_t + \mu_t\sigma_t} \right)^{1-\alpha} + \beta T_{t+1}^u$$

s.t.  $n + \mu_t m \geq (1+n)s_t + (1+m)\sigma_t \mu_t.$

If the decisive voter in the next period is controlled by the old, the utility to this

period's skilled young is

$$\begin{aligned}
V_t^{s|o} &= \max_{\sigma_t, \mu_t \in [0,1] \times [0,1]} V_t^s = \alpha^{1+\varepsilon\alpha} (1-\alpha)^{\varepsilon(1-\alpha)} A_t^{1+\varepsilon} \left( \frac{1-s_t + \mu_t(1-\sigma_t)}{s_t + \mu_t\sigma_t} \right)^{1-\alpha} + \beta T_{t+1}^o \\
\text{s.t.} \quad & 1 + \mu_t \geq (1+n)s_t + (1+m)\sigma_t\mu_t \\
\text{and} \quad & (1+n)s_t + (1+m)\sigma_t\mu_t \geq n + \mu_tm.
\end{aligned}$$

I denote with  $V_t^{s|u}$  and  $V_t^{s|o}$  the utility received by the skilled young given that, respectively, the unskilled young and the old form the largest group in the next period. Note that the constraints need not be binding. The solution to each problem could lie entirely in the interior of the constraint set. To conclude on the policy choices of the skilled young, they must be such that  $\langle \sigma_t^s, \mu_t^s \rangle = \arg \max \left\{ V_t^{s|u}, V_t^{s|o} \right\}$ .

Turning my attention to the old, their preference comes directly from the transfer.

$$\begin{aligned}
V_t^o &= T_t \\
&= \frac{\tau_t(1-\tau_t)^\varepsilon \alpha^{\varepsilon\alpha} (1-\alpha)^{\varepsilon(1-\alpha)} A_t^{1+\varepsilon}}{\left( 1 + \mu_t + \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \right)} (s_t + \mu_t\sigma_t)^\alpha (1-s_t + \mu_t(1-\sigma_t))^{1-\alpha}.
\end{aligned}$$

Optimization reveals that  $\tau_t^o = \frac{1}{1+\varepsilon}$ . In addition, I find that the interior optimal composition of skilled immigrants is given by

$$\sigma_t^o = \begin{cases} 0 & , \text{ if } \alpha < \frac{s}{1+\mu_t} \\ \frac{\alpha(1+\mu_t)-s_t}{\mu_t} & , \text{ if } \alpha \in \left[ \frac{s}{1+\mu_t}, \frac{s+\mu_t}{1+\mu_t} \right] \\ 1 & , \text{ if } \frac{s+\mu_t}{1+\mu_t} < \alpha. \end{cases}$$

Under my maintained assumption that the skilled wage is higher than unskilled wage, I have  $\sigma_t^o = 1$ . The optimal level of immigration volume,  $\mu_t$ , leads to the first-order condition

$$\left( \frac{\alpha\sigma}{s + \sigma\mu} + \frac{(1-\alpha)(1-\sigma)}{1-s + \mu(1-\sigma)} \right) \left( 1 + \mu_t + \frac{1 + \mu_{t-1}}{1+n + \mu_{t-1}(1+m)} \right) \gtrless 1.$$

Using the assumption that the wage is higher for the skilled than the unskilled workers, and imposing  $\sigma_t^o = 1$ , I can conclude that  $\mu_t^o = 1$ .  $\square$

## A.2 The Largest Group: Some Accounting

It is important to see who forms the largest political group in the economy and under what conditions. Remember that the largest group only needs to have its size bigger than the other groups separately, but not necessary collectively.

- **Skilled workers are the largest group** under two conditions. First, its size must dominate the unskilled young, and, second, it must also dominate the old cohort. Algebraically, these are  $s_t > \frac{1}{2}$  and  $s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$ , respectively. It can be shown that, under parameter restrictions above, the second condition of dominating the old cohort is sufficient.<sup>1</sup>
- **Unskilled workers are the largest group** under two conditions: dominating the size of the skilled young and the old cohort. These conditions are given respectively by  $s_t \leq \frac{1}{2}$  and  $1 - s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$ . Again, with the restrictions on parameters above, satisfying the second dominance is sufficient.
- **Old retirees are the largest group** when its size is larger than both skilled and unskilled young. This means

$$\frac{1 + \mu_{t-1}}{1 + n + \mu_{t-1}(1 + m)} \geq \max\{s_t, 1 - s_t\}.$$

---

<sup>1</sup>To see this, using  $n < m \leq 1$  and  $\mu \in [0, 1]$ , I can get the following string of inequalities:

$$\frac{1 + \mu}{1 + n + \mu(1 + m)} > \frac{1 + \mu}{(1 + \mu)(1 + m)} > \frac{1}{2}.$$

So if the skilled could dominate the old cohort, they automatically dominate the unskilled cohort.

### A.3 Today's Immigration Policies and Tomorrow's Largest Group

This subsection takes a look at how the choice of immigration policy variables in period  $t$  leads to a change in the identity of the largest group in period  $t+1$ . Recall that the model's skill dynamics are driven by the following equation

$$s_{t+1} = \frac{(1+n)s_t + (1+m)\sigma_t\mu_t}{1+n+\mu_t(1+m)}.$$

I assume weak inequality throughout, that is, if the decisive voters in this period try to influence the identity of the largest group in the next period, they will get their way as long as they make it to the threshold. This assumption is to ensure the existence of an optimum, by making the constraint set compact.

- For the skilled young to be the largest in period  $t+1$ , I need two conditions:  $s_{t+1} \geq \frac{1}{2}$  and  $s_{t+1} \geq \frac{1+\mu_t}{1+n+\mu_t(1+m)}$ . The first inequality is redundant (as previously discussed), so only the second equality plays a roll here. Therefore, using the skill dynamics, the condition for the skilled young to be the largest group boils down to selecting  $\sigma_t$  and  $\mu_t$  to satisfy  $(1+n)s_t + (1+m)\sigma_t\mu_t \geq 1 + \mu_t$ . This inequality gives rise to two cases, depending on the state of the economy.

(i) When  $s_t < \frac{1}{1+n}$ , I have  $\mu_t((1+m)\sigma_t - 1) \geq 1 - (1+n)s_t$ .

(ii) When  $s_t > \frac{1}{1+n}$ , I have  $(1+n)s_t - 1 \geq \mu_t(1 - (1+m)\sigma_t)$ .

- For the unskilled young to be the largest group in period  $t+1$ , I need two conditions:  $1 - s_{t+1} \geq \frac{1}{2}$  and  $1 - s_{t+1} \geq \frac{1+\mu_t}{1+n+\mu_t(1+m)}$ . Again, I only need to focus on the second inequality, which leads to  $(1+n)(1 - s_t) + (1+m)(1 -$

$\sigma_t)\mu_t \geq 1 + \mu_t$ . I also have two possible cases, depending on the state of the economy.

(i) When  $s_t < \frac{n}{1+n}$ , I have  $n - (1+n)s_t \geq \mu_t((1+m)\sigma_t - m)$ .

(ii) When  $s_t > \frac{n}{1+n}$ , I have  $\mu_t(m - (1+m)\sigma_t) \geq (1+n)s_t - n$ .

- For the old to dominate in size in period  $t + 1$ , I need two conditions:

$\frac{1+\mu_t}{1+n+\mu_t(1+m)} \geq s_{t+1}$  and  $\frac{1+\mu_t}{1+n+\mu_t(1+m)} \geq 1 - s_{t+1}$ , dominating the skilled and the unskilled, respectively. Both of these inequalities are important for crafting the right immigration policy to achieve the desired identity of the largest group. I have three cases to consider.

(i) When  $s_t < \frac{n}{1+n}$ , the old must

1. dominate the skilled,  $\mu_t((1+m)\sigma_t - 1) \leq 1 - (1+n)s_t$ , and
2. dominate the unskilled,  $n - (1+n)s_t \leq \mu_t((1+m)\sigma_t - m)$ .

(ii) When  $s_t \in [\frac{n}{1+n}, \frac{1}{1+n}]$ , the old must

1. dominate the skilled,  $\mu_t((1+m)\sigma_t - 1) \leq 1 - (1+n)s_t$ , and
2. dominate the unskilled,  $\mu_t(m - (1+m)\sigma_t) \leq (1+n)s_t - n$ .

(iii) When  $s_t > \frac{1}{1+n}$ , the old must

1. dominate the skilled,  $(1+n)s_t - 1 \leq \mu_t(1 - (1+m)\sigma_t)$ , and
2. dominate the unskilled,  $\mu_t(m - (1+m)\sigma_t) \leq (1+n)s_t - n$ .

APPENDIX B  
APPENDIX TO CHAPTER 2

## B.1 Proofs

*Proof. (Corollary 9)* I will show that  $\ln(R_{t+1})$  is increasing in  $\mu_t$  under the given condition. To do so, I use  $n_t = 1 + n + \mu_t(1 + m)$ , and differentiate the expression (2.21) with respect to  $\mu_t$  to obtain

$$\frac{\partial \ln R_{t+1}}{\partial \mu_t} = \frac{-(1 - \alpha)(1 - \alpha\phi)\gamma}{1 + \gamma\mu_t} + \frac{(1 + m)(1 - \alpha)}{1 + n + \mu_t(1 + m)} > 0.$$

To have positive response on the rate of return, I must have

$$\gamma < \frac{1 + m}{(1 + n)(1 - \alpha\phi) - \alpha\phi\mu_t(1 + m)}$$

which must hold for all  $\mu_t$ . Therefore, it is sufficient to have it hold for the lowest value of  $\mu_t$ , implying that  $\gamma < \frac{1+m}{(1-n)(1-\alpha\phi)}$ . With assumed parameter restrictions, this inequality always holds.  $\square$

*Proof. (Proposition 11)* Assume for the purpose of this proof that  $n < 0 < m$ . The other cases of these parameters are straightforward to see as no generation has influence over the identity of the decisive voter in the next period. Then for  $v(\mu_{t-1}) \geq 1$ , old generation forms the decision and hence will choose the maximal openness, that is  $\mu_t = 1$ . This follows directly from their preference, which I restate below

$$V_t^o(\mu_t, k_t) = (1 - \alpha) \ln \left( \frac{1 + \gamma\mu_t}{k_t} \right) + \ln w_{t-1} - \ln q_{t-1} + B^o \quad (\text{B.1})$$

$$\begin{aligned} V_t^y(\mu_t, \mu_{t+1}, k_t) &= (1 + \beta(1 - \phi(1 - \alpha)))\alpha \ln k_t + \beta(1 - \alpha) [\ln n_t + \ln(1 + \gamma\mu_{t+1})] \\ &\quad - ((1 + \beta)\alpha + \beta(1 - \alpha)(1 - \alpha\phi)) \ln(1 + \gamma\mu_t) + B^y. \end{aligned} \quad (\text{B.2})$$



When  $v(\mu_{t-1}) < 1$ , the decision belongs to the young. The ideal migration rate of the young is found by maximizing equation (B.2) with respect to  $\mu_t$ , holding other things constant. This ideal point is given by

$$\mu^* = \frac{\beta(1-\alpha)(1+m) - (1+n)\gamma[(1+\beta)\alpha + \beta(1-\alpha)(1-\alpha\phi)]}{\gamma\alpha(1+\beta[1-(1-\alpha)\phi])},$$

provided that  $0 \leq \mu^* \leq 1$ . Non-negativity constraint requires that  $\gamma < \underline{\gamma}$  where  $\underline{\gamma} = \frac{\beta(1-\alpha)(1+m)}{(1+n)(1+\beta[1-(1-\alpha)\phi])}$ . I will not ponder over the constraint  $\mu^* \leq 1$ , since it makes less sense for the young to have ideal migration rate higher or equal to that of the old who simply receives all benefits and incurs no cost. Before proceeding, I note that, regardless of the choice of migration today, the young always prefer next period's choice of migration quota to be one, that is  $\mu_{t+1} = 1$  (since they will be the old generation of that period). Furthermore, to ensure himself as the decisive voter in the next period, the young must consider the migration rate such that  $\frac{1+\mu_t}{1+n+\mu_t(1+m)} \geq 1$ , which means that  $\mu_t \leq -\frac{n}{m}$ .

Now suppose that  $\mu^* < -\frac{n}{m}$ , I now have to show that it is best for the young to just follow their ideal policy choice. Under this scenario, by choosing  $\mu^*$ , the young guarantees himself the position as the decisive voter in the next period as well as maximizing his current utility. Therefore, this is the most preferable scenario for the young, and he will get to choose his ideal policy positions in both periods of his life.

The case where  $\mu^* \geq -\frac{n}{m}$  is more complex. I must show that the young is willing to restrict some migration in order to place himself the decisive voter in the next period, when the incentive is right. Algebraically, this boils down to showing that

$$\begin{aligned} & -B \ln(1 + \gamma\mu^*) + \beta(1-\alpha) [\ln(1+n+\mu^*(1+m)) + \ln(1+\gamma\mu^*)] \\ < & -B \ln(1 - \gamma\frac{n}{m}) + \beta(1-\alpha) \left[ \ln(1+n - \frac{n}{m}(1+m)) + \ln(1+\gamma) \right] \end{aligned}$$

where  $B = (1 + \beta)\alpha + \beta(1 - \alpha)(1 - \alpha\phi)$  and I have substituted in all the relevant policy variables. With a little rearrangement, it is equivalent to showing

$$B \ln \left( \frac{1 + \gamma\mu^*}{1 - \gamma\frac{n}{m}} \right) + \beta(1 - \alpha) \left[ \ln \left( \frac{1 + n - \frac{n}{m}(1 + m)}{1 + n + \mu^*(1 + m)} \right) + \ln \left( \frac{1 + \gamma}{1 + \gamma\mu^*} \right) \right] > 0.$$

Written another way, I can define  $\Psi(\mu)$  as follows

$$\Psi(\mu) \equiv \left( \frac{1 + \gamma\mu}{1 - \gamma\frac{n}{m}} \right)^{(1+\beta)\alpha + \beta(1-\alpha)(1-\alpha\phi)} \left( \frac{1 + \gamma}{1 + \gamma\mu} \right)^{\beta(1-\alpha)} \left( \frac{1 + n - \frac{n}{m}(1 + m)}{1 + n + \mu(1 + m)} \right)^{\beta(1-\alpha)},$$

then I simply want the case where  $\Psi(\mu^*) > 1$ . Is this the case? Not necessarily. Notice that by the string of inequalities  $0 < -\frac{n}{m} < \mu^* < 1$ , the first two quantities of  $\Psi(\mu^*)$  are always greater than unity. So the question is whether the last quantity, the ratio of the population gross growth rates (which is less than one), is small enough to offset the other two. Under one setting this would not happen. When  $-\frac{n}{m}$  and  $\mu^*$  are close together (especially more relative to  $\mu^*$  and 1), the current ideal point and the strategic point are not too far apart, so it is worth the sacrifice to give up the ideal  $\mu^*$  in order to maintain decisive decision next period.

Consider now the case where  $\gamma \geq \underline{\gamma}$ , the ideal migration rate for the young now is zero, because the cost of having more immigrants outweighs the benefits that the young could reap (recall that I restrict the migration rate to be nonnegative, so there's no deportation). In this case, the young will be maximizing his payoffs in both period, since he again will be the majority next period (by default  $0 < -\frac{n}{m}$ ). This completes the proof.

For convenience of reference, I briefly summarize the necessary parameter values

and function used in writing the proposition below

$$\mu^* = \frac{\beta(1-\alpha)(1+m) - (1+n)\gamma[(1+\beta)\alpha + \beta(1-\alpha)(1-\alpha\phi)]}{(1+m)\gamma\alpha(1+\beta[1-(1-\alpha)\phi])} \quad (\text{B.3})$$

$$\underline{\gamma} = \frac{\beta(1-\alpha)(1+m)}{(1+n)(1+\beta[1-(1-\alpha)\phi])} \quad (\text{B.4})$$

$$\Psi(\mu) = \left( \frac{1+\gamma\mu}{1-\gamma\frac{n}{m}} \right)^{(1+\beta)\alpha+\beta(1-\alpha)(1-\alpha\phi)} \left[ \left( \frac{1+\gamma}{1+\gamma\mu} \right) \left( \frac{1+n-\frac{n}{m}(1+m)}{1+n+\mu(1+m)} \right) \right]^{\beta(1-\alpha)} \quad (\text{B.5})$$

□

*Proof.* (**Lemma 14**) I define formally the level  $\mu^*$  as (removing the time subscripts on immigrant's population growth rate because they are independent over time),

$$\mu^* = \frac{\beta(1-\alpha)(1+\mathbb{E}m) - (1+n)\gamma[(1+\beta)\alpha + \beta(1-\alpha)(1-\alpha\phi)]}{(1+\mathbb{E}m)\gamma\alpha(1+\beta[1-(1-\alpha)\phi])},$$

which should be compared with equation (B.3) to see that they are similar. Define the function

$$g(m, \mu) \equiv \frac{(1+\gamma\mu)(1+m)}{1+n+\mu(1+m)}.$$

Then I know that

$$\mathbb{E} \{g(m, \hat{\mu})\} = g(\mathbb{E}m, \mu^*)$$

because these two quantities equate to the same constant (see the first order condition). It can be shown that holding  $\mu$  constant,  $\frac{\partial^2 g(m, \mu)}{\partial m^2} \leq 0$ . So it must follow that

$$\begin{aligned} g(\mathbb{E}m, \hat{\mu}) &\geq \mathbb{E} \{g(m, \hat{\mu})\} \\ &= g(\mathbb{E}m, \mu^*) \end{aligned}$$

by Jensen's inequality. Since I assume that  $n < \mathbb{E}m$ , it follows that  $\gamma < \frac{1+\mathbb{E}m}{1+n}$ , which implies that  $\frac{\partial g(m, \mu)}{\partial \mu} < 0$ . With the above inequality, I can conclude that  $\mu^* \geq \hat{\mu}$ . □

*Proof.* (**Proposition 15**) Conditional on the belief parameter  $\zeta$ , I must show that it is optimal for the young to choose such a strategy. Note that, for if  $\zeta \leq \hat{\mu}$ , then for every immigration level that the young could choose to be strategic (i.e. within the set  $[0, \zeta]$ ), the young will be better off choosing  $\zeta$  since it's the closest to his bliss point. And if  $\zeta > \hat{\mu}$ , then the young will lead a happy life. He can choose his ideal policy  $\hat{\mu}$ , while believing that their cohort will continue to dominate the political arena in the next period. So I only need to focus on the question whether or not the young representative voter will want to act strategically. They will act strategically if

$$0 < [\beta + \alpha [1 - (1 - \alpha)\beta\phi]] \ln\left(\frac{1 + \gamma\hat{\mu}}{1 + \gamma\zeta}\right) + \beta(1 - \alpha)\mathbb{E}_{t-1} \left\{ \ln\left(\frac{1 + n + \zeta(1 + m)}{1 + n + \hat{\mu}(1 + m)}\right) \right\} \\ + \beta(1 - \alpha) \ln\left(\frac{1 + \gamma}{1 + \gamma\hat{\mu}}\right).$$

I use the fact that immigration in the next period will either be  $\hat{\mu}$  or 1. I do not need to consider the case where the next period's young decisive voter will act strategically, because if it is best for this period's young to be non-strategic, it will also be optimal for the next period's young to choose similarly (by stationary Markov-perfect property). So I can define the function

$$\hat{\Psi}(\mu, \zeta) = [\beta + \alpha [1 - (1 - \alpha)\beta\phi]] \ln\left(\frac{1 + \gamma\mu}{1 + \gamma\zeta}\right) \\ + \beta(1 - \alpha)\mathbb{E}_{t-1} \left\{ \ln\left(\frac{1 + n + \zeta(1 + m)}{1 + n + \mu(1 + m)}\right) \right\} + \beta(1 - \alpha) \ln\left(\frac{1 + \gamma}{1 + \gamma\mu}\right).$$

And the condition can be summarized as follows: the young decisive voter will act strategically if and only if  $\hat{\Psi}(\hat{\mu}, \zeta) > 0$  for any  $\zeta \in \left[0, -\frac{n}{\mathbb{E}_{t-1}m_t}\right]$ . And because negative level of labor migration is impermissible, so even if  $\hat{\mu} < \zeta$ , the young decisive voter can only choose as low as zero, which may be larger than  $\hat{\mu}$ .  $\square$